The Metrological Principle of Reciprocal Adjustment as used in the Great Pyramid of Giza

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ABSTRACT:

A close examination of the survey data on the Great Pyramid reveals the metrological technique used to encode some of the intended metrics of the monument, and provides a possible explanation of why the walls in the King's Chamber needed to be dressed and polished. Here we detail the method and discuss how the numbers can be extracted in general using this re-discovered ancient technique.

1 Introduction

The whole of metrology can be seen to rest on a very old subject to which the Classical Greeks gave the name '*anthyphairesis*.' It is believed that much of important mathematical thought was inherited by the Greeks from Egypt, but from the many destructions of libraries, in the times before the Christian era, little survives to document those things that might have been learnt from there. Modern scholarship tends therefore to attribute most things to the Greeks themselves, albeit with suspicions here and there that their ideas were often not original, but sourced from the old world. It is clear, from the vast monuments built by the Ancient Egyptians that they had to have mastered the art of measurement and the skill of construction in stone. So, it's an easy guess to think any ancient methodology concerning measurement might have come from them.

One such method is found in the Great Pyramid, through careful analysis, yet the Greeks did not seem to know of it, even though they had the methodological apparatus at hand to have suggested it. This is the subject we shall deal with in this text. The word 'anthyphairesis' generally means something like 'alternating subtraction,' and refers to the algorithm that compares two measures of similar type, by continually subtracting the smaller measure from

the larger, until a unit of measure can be established that would allow both original larger and smaller measures to be represented as integral multiples of that unit found.

From anthyphairesis was born the subject of 'continued fractions,' where the algorithm of subtractions is more efficiently replaced by a sequence of divisions, when applied to rational or real numbers, to accomplish the same task; and for comparison of pure integer numbers the procedure came to be called Euclid's GCD Algorithm, that is, the method of finding the 'greatest common divisor' of two whole numbers.

The object of anthyphairesis, therefore, as applied to *measures* rather than pure *numbers*, is to *discover* the optimal 'unit of measure' to enable the ready representation and thus comparison of two or more measurements. In terms of lengths, this optimal unit is the 'longest length' that can be used to represent the two original lengths being compared as integral multiples of the unit, so that the smallest number of counts would be required to represent these lengths. Once such a unit is found, it is obvious that this unit itself could be subdivided into an integral number of parts to produce many other units of length that can also represent the lengths as integral multiples of the unit, but the counts required in those representations will always be greater than the optimal case.

This is somewhat different from how we approach the subject today, where we start by first standardizing some unit of measure, and then proceed to measure everything with that standard unit. The *unit* is our gold standard, and we fix it and employ it everywhere throughout our modern designs. As a result, it is a bit mind bending for modern scientists, at least initially, to grapple with the idea that something might be built with no standard unit of measurement, but instead let the geometric features of the structure determine the actual definition of the unit—akin to anthyphairesis—to be used in its various parts, changing that unit here and there as the geometry of the part requires. Almost every part of the Great Pyramid that seems to use a unit of the cubit, for example, seems to use a slightly different length standard for that cubit. This, however, tells us that the unit itself is not as important to the builder as something else that is being represented in the measurements. Well, a measurement has two primary attributes, 'the unit' and 'the count' of those units, and so if the unit is not the primary attribute the builder was concerned with, we are left to conclude that it is 'the count' of units that takes center stage in his design. The unit, itself, may be adjusted here and there to fit 'the count' required into some measured dimension in the geometry. The unit does carry a very important meaning. The cubit is the forearm of a man, for example. But, just as the forearm of a man can vary from one man to the next, the cubit's length can vary too, often times even within the same structure built under a particular Pharaoh.

We therefore have these two ways to approach measures used in construction. We can let the geometry of the design suggest the best definition of the

unit to fit into some part of the structure, letting that geometry tell us what is optimal, like anthyphairesis; or we can stamp a standard ruler with our one and only defined unit, and having fixed that, go about our design using it in all parts regardless of whether it is convenient or not to represent the dimensions we plan to construct. The first method is flexible in adapting the standard to fit the geometry; the second one adheres to rigid standards, forcing the geometry to fit the standard.

Now, there was a time when the Greeks believed that all pairs of lengths were commensurable, and could be represented therefore as multiples of some discoverable unit of length; and then they realized that the diagonal length of a square was not commensurable with its side length, and this threw a wrench into the method of 'anthyphairesis,' which now would not terminate in a finite number of steps to reveal that special optimal unit of length to be used. For these incommensurable lengths, however, the algorithm was still the best way to find that optimal unit of length limited to a particular required *precision*, by artificially terminating the infinite sequence to the appropriate finite number of steps. So, again, given a stated *precision* one could still establish an optimal unit of length within this newly required constraint.

2 The Reciprocal Theory

Let's take a brief look at how anthyphairesis, or continued fractions, might be applied to real valued numbers whose decimal representations are given. We'll apply this to π and ϕ, which are always good places to start.

	π	ϕ
$z :=$	3.14159265359	1.61803398875
$[z] =$	3	1
$z := z - [z] =$	0.14159265359	0.61803398875
$z := 1/z =$	7.06251330593	1.61803398875
$[z] =$	7	1
$z := z - [z] =$	0.06251330593	0.61803398875
$z := 1/z =$	15.9965944067	1.61803398875
$[z] =$	15	1
$z := z - [z] =$	0.99659440669	0.61803398875
$z := 1/z =$	1.00341723101	1.61803398875
$[z] =$	1	1
$z := z - [z] =$	0.00341723101	0.61803398875
$z := 1/z =$	292.634591014	1.61803398875
$[z] =$	292	1
$z := z - [z] =$	0.63459101440	0.61803398875
$z := 1/z =$	1.57581808963	1.61803398875
$[z] =$	1	1
$z := z - [z] =$	0.57581808963	0.61803398875
$z := 1/z =$	1.73665957706	1.61803398875
$[z] =$	1	1
$z := z - [z] =$	0.73665957706	0.61803398875
$z := 1/z =$	1.35747912758	1.61803398875
$[z] =$	1	1
$z := z - [z] =$	0.35747912758	0.61803398875
$z := 1/z =$	2.79736611969	1.61803398875
$[z] =$	2	1

$$\pi = [3; 7, 15, 1, 292, 1, 1, 1, 2, \cdots] \qquad \phi = [1; 1, 1, 1, 1, 1, 1, 1, 1, \cdots]$$

The algorithm here is simple: *truncate to nearest integer, subtract this integer from the value, take the reciprocal, and repeat.*

The objective is to collect the sequence of integers, $[z]$, that appear at each level, and terminate the sequence when some condition is reached; either because the sequence terminates naturally, as in rational numbers, or because we've reached the precision requirement set as the objective.

When we subtract the integer part, e.g. $3.14159265359 - 3 = .14159265359$, we get what is left over, that is, the error, missing from the value, and by taking this as our *new* 'unit of measure,' the subsequent taking of its reciprocal basically asks how many times does this divide the *previous* 'unit of measure.' We get $1/0.14159265359 = 7.06251330503$, so the error length goes into the diameter 7 times plus a bit. This tells us we can improve our *precision* by dividing the diameter into 7, and measuring the circle ratio $\pi = C/D$, as $(3\times7+1)/7 = 22/7$, instead of $3/1$, and so on. Incidentally, we see the snychronicity here with the "cubit" being divided into 7 palms, for when the cubit is used as the diameter, the circumference would be 3 cubits plus an error length. That error length is about 1 palm width, so that it divides the cubit into 7 parts, and so on. This is part of what we would call "sacred metrology," where the ratios employed in the metrological system encode relationships found in the natural world. This is as opposed to "common metrology," such as setting all ratios as multiples of 10, like in the metric system, which is done more for convenience of calculation, but doesn't incorporate any idea about the physical world that could be deduced and thus extracted back out from just looking at the measurement system.

With these integer sequences found, the values, π and ϕ, can be written as continued fractions,

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cdots}}}}}}}} \tag{1}$$

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}}}} \tag{2}$$

Now, the important thing we'd like to point out here is the remarkable difference between these sequences, and their corresponding continued fraction forms, as the ϕ ratio always returns the value '1' on each level, when the question is asked—by taking the reciprocal—"How many times does the *new* 'unit of measure' divide the *previous* 'unit of measure'?", in the hunt for the ultimate representation of the number in terms of some optimal unit.

This golden mean, ϕ, is so special, that it is often referred to as the most irrational number, since its continued fraction is the slowest to converge to a given level of precision, of all real numbers. Every real number has a continued fraction representation, but, unless that real number is ϕ itself, one or more of its integer elements in sequence will differ from the unit '1', and when it does, at that level it will converge faster than ϕ to its true value.

What we see from this, is that the special nature of ϕ, makes it singularly unique in the context of "*measurement*," since the algorithm typically used to *discover* the optimal unit of length, for example, always returns "the count" of '1', at each level of precision. Its entire sequence is made up of unit counts, in seeking the unit of length. In fact, the ϕ ratio has this famous formula relating its reciprocal to the unit:

$$\phi - \frac{1}{\phi} = 1 \tag{3}$$

This formula is also characteristic of the main features in the measurement algorithm, as it incorporates both 'the reciprocal' and 'the subtraction' that are key steps in the method of anthyphairesis in seeking that optimal unit '1'. Hence, at a glance, we can see the methodology reflected in the expression. So, anyone familiar with the ancient algorithm of measurement, and the ϕ ratio's various forms of expression and representation, will be naturally led to the next intuitive step in measurement, which is that it is possible to use these ideas from the algorithm, with a slight change, to make a constructed unit of measure *discoverable*, instead of applying an algorithm in attempt to *discover* the unit.

We'll refer to this next idea as "*The Principle of Reciprocal Adjustment*," in keeping with the general motif employed in steps used previously above in the standard measurement algorithm. It requires us to use a pair of similar lengths, L, L', to be compared, as usual, which we establish by deliberate construction, where the one length, L, is adjusted by a small amount via "*a reciprocal*" to yield the second slightly shorter length, L', defined as follows:

$$L = n \cdot u \tag{4}$$

$$L' = (n - \frac{1}{n}) \cdot u \tag{5}$$

$$L'/L = (1 - \frac{1}{n^2}) \tag{6}$$

$$n = 1/\sqrt{1 - L'/L} \tag{7}$$

$$u = \sqrt{L \cdot (L - L')} \tag{8}$$

> *Rather than taking the reciprocal of the "unit" to <u>decode</u> the "count" inherent in some measurable length, we instead take the reciprocal of the desired "count" to <u>encode</u> the "unit" in our constructed length.*

Instead of taking the reciprocal of 'the unit' this time, we employ the reciprocal of 'the count,' to enable the representation of the local metrological parameters, i.e. both 'the unit' (u) and 'the count' (n), in the dimensions of the structure. Then as the inverse formulas for these parameters show, n and u, can be determined by anyone inspecting the construction at a later time, by using any measurement system, whether it be cubits, inches, metres, or other, by simply calculating them using the measured back values. This makes *discoverable*, what the architect originally intended these parameters to be, as both 'the unit' (u), and 'the count' (n) used in the length, L, can be readily re-covered.

Metrology Example. Let's consider an example of how the builder could include information about the unit of length used, and the number of units applied in the design, when engaging the construction. Suppose the intention is to erect a rectangle platform (say, maybe a niche recess in a side wall intended to hold a religious artifact), with dimensions on the blueprint, $W \times L = 8'' \times 10''$. The architect is using the *imperial* metrological system, so measures in *inches*. To indicate the unit of the inch, and the count of 10 for the units used in the long side, L, the builder decides to employ *the principle of reciprocal adjustment*. So instead of making a perfect rectangle, $W \times L$, he lays out a distorted rectangle, morphed into a trapezoid, with 4 dimensions, W, L, W, L', where now the parallel side is, $L' = (10 - 1/10)''$, so that it is shortened just a bit by subtracting that reciprocal, $1/10$, of an inch.

Some time later, a modern French archaeologist shows up on the scene, and starts measuring everything with his new and improved metric system, and finds the results of the 4 sides, expressed in centimeters:

$$W = 20.320\,\text{cm}, \quad L = 25.400\,\text{cm}, \quad W = 20.320\,\text{cm}, \quad L' = 25.146\,\text{cm}.$$

Let's see how he may go about discovering the original units used in the construction of the niche platform. Luckily, this French archaeologist is aware of *the principle of reciprocal adjustment*. So when he notices that, what initially looks like an apparent rectangle to the eye, actually measures out to a trapezoid, instead of thinking it must be a construction error made by incompetent workmen, he posits that the builders were probably smarter and more skilled than that, and this variation from the perfect rectangle shape must be deliberate. Then, comparing the unknown original parameters, (n, u), for the count, n, and unit, u, to his measurements, he finds:

$$\begin{aligned} L &= n \cdot u &&= 25.400\,\text{cm} && (9) \\ L' &= (n - \frac{1}{n}) \cdot u &&= 25.146\,\text{cm} && (10) \end{aligned}$$

$$\frac{L'}{L} = (1 - \frac{1}{n^2}) = \frac{25.146}{25.400} = \frac{25.400 - 0.254}{25.400} = (1 - \frac{1}{100}) \qquad (11)$$

$\therefore$

$$\begin{aligned} n &= 10 && (12) \\ 1 \cdot u &= 2.54\,\text{cm} && (13) \end{aligned}$$

So, he concludes the builders of the platform must have used a unit of length equivalent to, $u = 2.54\,\text{cm}$, that 10 such units make up the length, L, and consequently, 8 units make up the width, W, so that the original design was nominally

intended to be, $W \times L = 8u \times 10u$; and the only thing our French archaeologist is missing, is the name of the unit, u, which those ancients called the "*inch*."

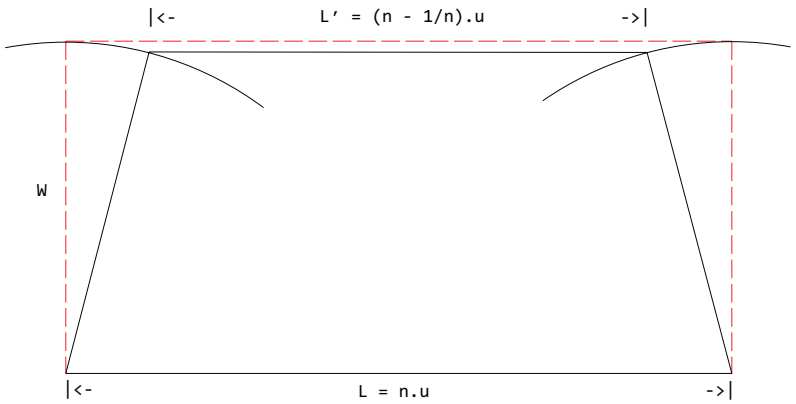

Figure 1: *The Principle of Reciprocal Adjustment applied to one side of the rectangle.*

The figure above shows a greatly exaggerated example of the distortion applied to the design rectangle during the construction phase in order to encode the unit of measure, u, used, and the number of such units, n, to measure out the length, L, in a rectangle of dimensions, $W \times L$. In practice, the actual distortion is typically too small and subtle to be observed by casual inspection with the eye, and a more detailed examination by actual measurement is required to reveal the distortion. Once the builder has measured out the parallel lines according to, $L = n \cdot u$, and, $L' = (n - 1/n) \cdot u$, then, using any other metrological system to re-measure the same pair, L, L', at some later time after construction, will allow for the re-discovery of the actual unit, $u = \sqrt{L(L - L')}$, and number of such units, $n = 1/\sqrt{1 - L'/L}$, that were originally used by the designer and builder of the works. The one caveat is that the small error introduced, u/n, must be significantly larger than the resolution, r, of the measuring instruments, $|L - L'| > r$, so that it could be clearly distinguished from what would otherwise be considered a normal precision error of construction; that is, ideally, the variance introduced would be too small to see by casual visual inspection, but large enough to be clearly and unambiguously detected by actual measurement. By casual inspection the design intent is implicated, e.g. it's a 'rectangle,' while actual measurement reveals the unit used and so enables an accurate determination of the intended 'count' of units, revealing the numbers encoded in the dimensions. The numbers and shapes can then convey a message.

Cubit Metrology. Now, the Royal Cubit Rod is divided into 7 palms, of 4 digits each, with a further subdivision of each digit into 16 parts; the *digit* representing the average *width* of the finger on the human hand. So, there are in all a total of $7 \times 4 \times 16 = 28 \times 16 = 7 \times 64 = 448$ divisions on the cubit rod. Using the measure of 20.632 inches reported by archaeologist Petrie, the *resolution* of the Royal Cubit rod is then: $r = 20.632''/448 = 0.046''$.

This means that to apply *the principle of reciprocal adjustment* to a length, using the *cubit* as the unit of length, in order for the deliberate deviation error introduced to be larger than the resolution of this measuring rod, $u/n > 0.046''$, there is an upper bound on the number of cubits the length can have, that is,

$n < u/0.046'' = 20.632''/0.046'' = 448$. So, the max constructable length is 448 cubits, which is on the order of the length of the base of the Great Pyramid, which measures right about 440 cubits in length. So, the Great Pyramid is constructed at or near the limit of this particular measuring tool. If you layout a length longer than 448 cubits, then it's not possible to include a *reference* to the unit used, i.e. the cubit, and the number of units, when $n > 448$, in the architecture, because the resolution is now larger than the reciprocal adjustment required to implicate the values of these parameters. Hence, the actual scale of the Great Pyramid and the dimensions of the Royal Cubit Rod measuring tool are intricately linked to each other. By using 440 cubits for a side length, the reciprocal deviation required is, $20.632''/440 \approx 0.047'' > 0.046''$, a difference of about $0.001''$, or one thousandths of an inch. This is just significantly larger than the resolution, to be unambiguously detected. Much smaller, and it could not be distinguished from a normal error of measurement using '*this*' measuring tool.

$$448\,cu \times 448\,cu$$

THE LIMIT SQUARE

In setting forth to design the Great Pyramid, therefore, one of the first things to be established is the "*limit square*" at the base on the Giza Plateau, within which all of the pyramid's design features would ultimately be required to fit. This square is $448\,cu \times 448\,cu$; a constraint established to indicate to the visitor that the layout of the monument is set according to 'the measure of a man,' and the divisions on the standard cubit of his arm. As we can see, the "*core square*," the "*casing square*," and the "*socket square*," at the pyramid base, are all contained within this "*limit square*," in a nested formation of increasing dimensions.

But it wouldn't be practical to actually expect the construction workers to lay down any line of 440 cubits to a precision of one thousandths of an inch within the resolution, using an actual cubit rod. Every time the worker lifts and places the cubit rod, human error would naturally introduce a small displacement in locating the additional points along the line, so that after 440 repetitions of such action, the accumulated error would be 440 times greater than that of one measurement. Even if the man could place the rod down each time making only $0.001''$ error, there would still be $440 \times 0.001'' = 0.44''$ error introduced in laying out the cubits required for one side of the pyramid base. So, apart from using some sophisticated advanced laser guided measurement system, the builders could not expect to construct the length of 440 cubits reliably to anything closer than about half an inch this way. This suggests to us that the base of the pyramid must be established using longer length units than the cubit itself; we shall explore the evidence of this below in the "socket square."

Tolerances of Construction and Re-Measurement.

Once the theoretical parameters have been set, $\{L, n, u\}$, the very next consideration is the tolerances required in construction to implement the plan, so that re-discovery of these values are possible in some unambiguous way. With the numeric count chosen, n, the reference length, L, and its apposite [1], L', are constructed to have the small difference;

$$\Delta L(n) \;=\; L - L' \;=\; \frac{1}{n}\cdot u \;=\; \frac{1}{n}\cdot(\frac{L}{n}) \;=\; \frac{L}{n^2} \tag{14}$$

and with the requirement that any future survey of the finished construction must be able to re-cover that count, n, the tolerances in the assembly must be such that the neighboring count values on either side, $n+1$ and $n-1$, must be clearly outside the range of errors that could be associated with both the construction effort, and the re-measurement by any future survey.

$$\Delta L(n) \;=\; L/n^2 \tag{15}$$
$$\Delta L(n+1) \;=\; L/(n+1)^2 \tag{16}$$
$$\Delta L(n-1) \;=\; L/(n-1)^2 \tag{17}$$

Let us consider this difference as a 'function' of 'n', i.e. $\Delta L(n)$, then the three values of concern automatically fit the ordered relation,

$$\Delta L(n+1) < \Delta L(n) < \Delta L(n-1) \tag{18}$$

Our target difference, $\Delta L(n)$, is separated from its neighbors above and below by the ranges, δ_+ and δ_-, respectively,

$$\delta_+ \;=\; \Delta L(n-1) - \Delta L(n) \tag{19}$$
$$\delta_- \;=\; \Delta L(n) - \Delta L(n+1) \tag{20}$$

When we measure the difference, therefore, $measurement = meas\,\langle\Delta L(n)\rangle$, we must have the value found to be within $1/2$ of the separation of the upper, and within $1/2$ of the separation of the lower, to be confident that we have found the intended, n, and not one of the nearby count values.

$$\Delta L(n) - \frac{1}{2}\cdot\delta_- < \text{meas}\,\langle\Delta L(n)\rangle < \Delta L(n) + \frac{1}{2}\cdot\delta_+ \tag{21}$$

[1] We have appropriated the term "*apposite*" here *to the purpose* of playing the role of counterpart, L', to reference length, L.

Hence, plugging in the expressions for these theoretical formulas, we obtain:

$$\frac{L}{n^2} - \frac{1}{2} \cdot \delta_- < \text{meas}\,\langle \Delta L(n) \rangle < \frac{L}{n^2} + \frac{1}{2} \cdot \delta_+ \tag{22}$$

with the ranges,

$$\delta_+ = \frac{L}{(n-1)^2} - \frac{L}{n^2} \tag{23}$$

$$\delta_- = \frac{L}{n^2} - \frac{L}{(n+1)^2} \tag{24}$$

thus we must ideally have the measured difference, $L - L'$, fit wholly within the range:

$$\frac{1}{2} \cdot \left(\frac{1}{n^2} + \frac{1}{(n+1)^2} \right) < \frac{\text{meas}\,\langle \Delta L(n) \rangle}{L} < \frac{1}{2} \cdot \left(\frac{1}{n^2} + \frac{1}{(n-1)^2} \right) \tag{25}$$

Now, if we knew the designer's true value for the reference length, L, and corresponding apposite, L', as indicated on the blueprints for the construction, we would be in a better position to evaluate the measurement, "meas $\langle \Delta L(n) \rangle / L$," with virtual certainty in the conclusion, having tested the measurement to fit this range condition. But, since we don't know the original, L or L', we have to 'estimate' these by substituting our measured values, $L \approx \text{meas}(L)$ and $L' \approx \text{meas}(L')$. So, the construction engineers have an advantage over the subsequent surveyors, in that in placing the parts of the building to fit the design, they know the true values of the pair, $\{L, L'\}$, and they have the option to aim for the center of the range in setting construction within tolerances, so that the future surveyor would be more likely to find the right values for the parameters within the range of his own measurement errors.

What we shall do here, in analysing the survey data available to us, is set the reference length, L, to the measured value, and assume all error of construction and of survey re-measurement, is attributed to the apposite, L'; and where we have to take into account the tolerance required in the surface of walls, we will divide that total tolerance value by 2, to attribute half to one wall surface and half to the other opposite wall surface, in order to estimate the flatness required in grounding down and polishing those walls to meet the specification.

3 The King's Chamber

As remarked by almost all of the surveyors who visited the Great Pyramid and made attempts to measure its various parts, the King's Chamber is unquestionably the most well constructed feature, with an obvious meticulous attention to detail in the layout of this room. So, we have very good measurements of the dimensions of the inside of the King's Chamber, and these numbers appear to be known with sufficient precision to enable us to extract the relevant information from there with our *Reciprocal Theory*. So, let's look at these numbers first. In his 1883 book "*The Pyramids and Temples of Gizeh*," archaeologist W. M. Flinders Petrie publishes the measures for length, width, and height, of the King's Chamber. For the present discussion, we only need 8 measured values from Petrie's table; the width on the top and bottom of the East and West walls, and the length on the top and bottom of the North and South Walls of the room.

	N	E	S	W
Top	412.14″	206.30″	411.88″	206.04″
Base	412.78″	206.43″	412.53″	206.16″

`The King's Chamber dimensions given by Petrie in 1883.`

In principle, a rectangular box has 6 faces, and 12 lines, bounding these faces, that thus provide 12-pairs of opposite nominally parallel lines that could be used for analysis, where each pair of parallel lines being compared fall on the same face (i.e. diagonally opposite parallels excluded, which would otherwise add another 6 pairs, making the count 18-pairs). But, in the case of the King's Chamber, only the rectangles in the horizontal plane are of interest to us at the moment. So, only the 4 line measures on the perimeter of the floor, and 4 line measures on the perimeter of the ceiling, are of use here. The reason we don't use the vertical lines, is that the King's Chamber is only "polished" on the vertical surfaces, leaving the floor and roof in rough cut unpolished stone (the roof is dressed flat, but unpolished; while the floor is in undressed rough cut state). So, any vertical line from floor to roof would give a different measurement depending on where the plumb line was set up, and only the horizontal lines can be guaranteed to reflect the deliberate intention of the architect. Any variation found in lengths of the horizontal line would indicate deliberate intention, while the variations in vertical lines would just reflect the random nature of the rough cut surface of the granite stone. The architect removed this rough cut "noise" in the horizontal dimensions, by carefully polishing the granite stones in the vertical walls, to eliminate any error of measurement introduced by the natural variations in the stone. In this way, the designer could then encode a deliberate "*systematic*" variation in the parallels, that would be obvious, and not hampered by the randomness of rough stone surfaces.

	A	B	C	D
	East Wall	West Wall	$\boldsymbol{W = n \cdot u}$	$W = \frac{n}{2} \cdot u$
			$W' = (n - \frac{1}{n}) \cdot u$	$W' = (\frac{n}{2} - \frac{1}{n}) \cdot u$
			.: n = ..	.: n = ..
	W	W'		
Top	$206.30''$	$206.04''$	28.168	39.836
Bottom	$206.43''$	$206.16''$	27.651	39.104

	A	B	C	D
	North Wall	South Wall	$L = 2n \cdot u$	$\boldsymbol{L = n \cdot u}$
			$L' = (2n - \frac{1}{n}) \cdot u$	$L' = (n - \frac{1}{n}) \cdot u$
			.: n = ..	.: n = ..
	L	L'		
Top	$412.14''$	$411.88''$	28.153	39.814
Bottom	$412.78''$	$412.53''$	28.733	40.634

The first two columns, A and B, contain the raw data from Petrie's table. The remaining columns, C and D, show the results of the reciprocal adjustment applied to this survey data. In column, C, the East side of the room is taken as the start of the measurement, the reference from which all other measurements are determined; so the "width" of the room, $W = n \cdot u$, is the starting point for measurement. In column, D, the North side of the room is taken as the start and reference instead; thus, it is the "length" of the room, $L = n \cdot u$, which is considered the start point for measuring. We look at both of these two approaches, because both the East and the North sides are longer than the corresponding nominal parallel on the other side of the room.

The Quarter Inch. Using the longer East wall as the reference, and comparing the shorter West wall, we have the width of the room at the ceiling level differs by, $206.30 - 206.04 = 0.26$ inches, while the difference at the floor level is, $206.43 - 206.16 = 0.27$ inches. When using the longer North wall instead as the reference, and comparing the shorter South wall, we have the length of the room at the ceiling level differ by, $412.14 - 411.88 = 0.26$ inches, and the difference at floor level is, $412.78 - 412.53 = 0.25$ inches. In all four cases, the difference is about $\frac{1}{4}$ of an inch. For this reason, the width, W, and the length, L, appear at first to be inseparable from each other. The $\frac{1}{4}$ inch error seems independent of the dimension. So, if we pick the East wall width, $W = n \cdot u$, as the reference,

then, although the North wall length is nominally double the width, $L = 2n \cdot u$, the South wall must be assigned the same absolute error, u/n, as the West wall, so, $L' = (2n - 1/n) \cdot u$, and $W' = (n - 1/n) \cdot u$. Similarly, when we start with the North wall length as the reference, $L = n \cdot u$, then the same absolute value of the reciprocal error, u/n, must be used for both South and West walls, even though the East wall width is now nominally half that of the North, $W = n/2 \cdot u$.

The Results. The essential results from the four data points in column, C, for the width, W, and the four data points in column, D, for the length, L, are as follows;

$$W: \quad (28.168 + 27.651 + 28.153 + 28.733)/4 \quad = \quad 28.176 \quad = \quad 28 + .176 \quad \Longrightarrow \quad n = 28 \tag{26}$$
$$L: \quad (39.814 + 40.634 + 39.836 + 39.104)/4 \quad = \quad 39.847 \quad = \quad 40 - .153 \quad \Longrightarrow \quad n = 40 \tag{27}$$

$$40/\sqrt{2} \quad = \quad 28.284 \quad = \quad 28 + .284 \qquad \text{note: } .176 < .284 \tag{28}$$
$$28 \times \sqrt{2} \quad = \quad 39.598 \quad = \quad 40 - .402 \qquad \text{note: } .153 < .402 \tag{29}$$

The closest whole number values for the number of units, n, used in laying out the wall, is then, $n = 28$, for the width, W, and, $n = 40$, for the length, L. It is actually not possible to obtain both, $W = 28 \cdot u'$ and $L = 40 \cdot u''$, using this convention, since the common reciprocal error ties the two numbers together by a relation involving the $\sqrt{2}$, as shown above. We shall see why in a moment. For now, we just want to note that the architect manages to get results much closer to the whole number values, 28 and 40, than would be possible by setting one exactly to either, 28 or 40, then relying on the construction to implicate the other alternate. We compute the $\sqrt{2}$ correspondences above to illustrate. The architect achieves this closer result by using 4 measures instead of the 2 minimal measures required to implicate one parameter pair (n, u). This allows him to jiggle the numbers in the 8 candidates for measured lengths, to better fit the objective of simultaneously encoding two different units of length, (u', u''), in the room.

$$W \times L \quad = \quad 28u' \times 56u' \tag{30}$$
$$W \times L \quad = \quad 20u'' \times 40u'' \tag{31}$$

$$28 \quad = \quad 2 \cdot 2 \cdot 7 \tag{32}$$
$$20 \quad = \quad 2 \cdot 2 \cdot 5 \tag{33}$$
$$140 \quad = \quad 2 \cdot 2 \cdot 5 \cdot 7 \qquad \textbf{LCM(20,28) = 140} \tag{34}$$
$$280 \quad = \quad 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \tag{35}$$

$$W \times L \quad = \quad 140u \times 280u \tag{36}$$
$$= \quad 28 \cdot (5u) \times 56 \cdot (5u) \quad \Longrightarrow \quad u' = 5u \tag{37}$$
$$= \quad 20 \cdot (7u) \times 40 \cdot (7u) \quad \Longrightarrow \quad u'' = 7u \tag{38}$$

Starting with the East Wall, then, we find the suggestion that the room has dimensions, $W \times L = 28u' \times 56u'$, while starting with the North Wall, the

suggestion is that the room measures, $W \times L = 20u'' \times 40u''$. The least common multiple of the numbers for the Width, 20 and 28, is 140. This means that for both of these metrological systems to co-exist simultaneously, there must be another smaller unit, u, common to both systems, so that, $u' = 5u$, and, $u'' = 7u$. Then, the room measures, $W \times L = 140u \times 280u$, on this lower scale.

The Two Fingers. Given that we know the room is nominally 10 cubits by 20 cubits, and the cubit is 28 digits, where the digit, d, is the nominal width of a finger on the human hand, making the dimensions then also, $W \times L = 280d \times 560d$, that unit, u, we just found in the lower scale, is then equivalent to 2 digits, $u = 2d$, or the width of two fingers held together as shown in the following hand gesture below. But, notice that, even if we didn't have any idea of the measure of the "cubit" in mind, and/or knowledge of its 28 parts, and so on, we would still observe that the unit, u, here found, is about the width of two fingers by checking our own hand. So, without any foreknowledge of the existence of a unit like the cubit, once we notice this unit, u, measures the width of two fingers on our own hand, $u = 2d$, we would immediately recover the dimensions of the room, $W \times L = 280d \times 560d$, as measured by our body parts, by simple observation, measurement, and reflection.

So, then the question is: *why two fingers* ?

A few Egyptologists will probably jump at this as proof that the King's Chamber was in fact a burial chamber, as they have claimed all along, since Egyptian mummies were often found with 'two-finger amulets' wrapped up with the dead. The numeric symbolism thus seems to match the physical ornament symbolism usually found in a tomb. But, we shall deal with the thorough interpretation of these numbers in a future work. For now, our focus is just in describing the technical mechanics of extracting the numerical parameters from the survey data. Something which can be readily 'verified' or 'falsified' by the scientific minded surveyor. What the numbers mean, and why they were put there by the architect, can be considered somewhat more subjective, and open to interpretation, and fall outside the scope of things subject to scientific verification; so these issues are better left for a more expansive work that draws on a wider collection of sources for supporting evidence on any claims. What we can say here, without requiring any reference source, is the one obvious interpretation that seems self-evident.

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Figure 2: *Hand Gesture Two Fingers*

The width of two fingers held together forms the ultimate unit of measure, $u = 2d$, found encoded in the walls of the King's Chamber. The gesture of the index and medial fingers held together is a natural symbol for the idea "to measure," or of "measurement." Then, taking the length of the longer middle finger as reference, L, and the length shorter index finger as apposite, L', measure and calculate the count, $n = 1/\sqrt{1 - L'/L}$, to verify, $n = 3$, is the best fit for the number of segments on a finger; and thus confirm that the metrological principle of reciprocal adjustment is inherently encoded in the human hand!

The index and middle fingers held together:

The "index finger" is used for counting. We naturally and automatically *point* with the index finger to the things being counted, and say, 1..2..3..etc..moving that finger to indicate each item in turn. So, index is for enumeration. In fact, the word "index" itself, refers to that very idea of assigning an integer number to a thing, as in counting.

Conversely, the "middle finger" is used in the definition of the unit of length, that is, the cubit is the length of the forearm from elbow to tip of that particular "middle finger." Again, this is natural and automatic, given that this is the longest finger in the hand, so marks the extreme end of the forearm when establishing that distance from elbow to end of the hand as the unit of length.

So, together, these are the two key fingers involved in counting and measuring. Held together, the gesture then refers to the concept of "measurement," just as we "count" the number of "units of length" to determine "the measurement." That is to say, the operation of measurement involves the *combination* of the two very things attributed to each of these two particular fingers.

We always need both a "unit" and a "count" in any measurement. The two-finger symbol can then be interpreted as the hint "to measure," or "measure this," or "notice the measurement of," etc.. The use of the two-finger unit in the dimensions of the room, then suggests we measure the room and things within.

Or rather, since we only actually *discover* the two-finger unit after we have already measured the room, it would mean then, in that context, that we should "pay attention to the measurements" that we have uncovered. Finding the two-finger width unit in the King's Chamber suggests the architect wanted the viewer to carefully measure the dimensions of everything within and contemplate those measurements. Which again, suggests that hidden in the measurements is a message we have to decipher.

Figure 3: *The two-fingers hieroglyph.*

Apart from "two-finger amulets" appearing in tombs of the deceased, the Egyptian Hieroglyphs also included a similar "two-finger glyph" used as the determinative in the words "straightforward," and "testify," and "exact," and "precise," and "accurate," as discussed by *Gardiner*, and *Budge*, *et al.* The way we get "accurate" and "precise," etc... of course, is through "measurement."

Analytic Considerations. Now, let us see what is the problem attempting to encode two different units, u', u'', simultaneously, in the same side of a rectangle, with only one value for the measure of the reciprocal error, like the $0.26''$ present here. Suppose we have a rectangle that is p times as long as it is wide, so, $W \times L = w \times pw$. Let the distortion applied to this rectangle produce the shorter sides, W' and L', and let, (n, m) be the two 'counts' required when using the two units, (u', u''); first taking the longer Width, $W = n \cdot u'$, for reference, then taking the longer Length, $L = m \cdot u''$, for reference.

East	Wall	North	Wall
W	$= n \cdot u'$	W	$= \frac{m}{p} \cdot u''$
W'	$= (n - \frac{1}{n}) \cdot u'$	W'	$= (\frac{m}{p} - \frac{1}{m}) \cdot u''$
L	$= pn \cdot u'$	L	$= m \cdot u''$
L'	$= (pn - \frac{1}{n}) \cdot u'$	L'	$= (m - \frac{1}{m}) \cdot u''$

Equating the two models:

$$\begin{aligned} n \cdot u' &= \frac{m}{p} \cdot u'' \\ (n - \frac{1}{n}) \cdot u' &= (\frac{m}{p} - \frac{1}{m}) \cdot u'' \\ pn \cdot u' &= m \cdot u'' \\ (pn - \frac{1}{n}) \cdot u' &= (m - \frac{1}{m}) \cdot u'' \end{aligned}$$

$$\therefore \quad p = \frac{m^2}{n^2} \ , \quad n = \frac{m}{\sqrt{p}} \ , \ m = n \cdot \sqrt{p} \tag{39}$$

When p is a perfect square, $p \in \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \cdots\}$, then the ratio, m/n, is a whole number. So, then we can easily have two units, (u', u''), simultaneously defined and encoded, with a whole number count, n, for number of units that make up the Width, and a whole number count, m, for number of units that make up the Length. But, in our case, the room is twice as long as it is wide, so, $p = 2$. Hence, if we set the Width count, $W = 28 \cdot u'$, then the Length count is irrational, $L = 28 \cdot \sqrt{2} \cdot u'' = 39.597 \cdot u'' \neq 40 \cdot u''$, and if we set the Length count, $L = 40 \cdot u''$, then the Width count is irrational, $W = 40/\sqrt{2} \cdot u' = 28.284 \cdot u' \neq 28 \cdot u'$. The two units, (u', u''), are related by the formula:

$$u'' = \frac{m}{n} \cdot u' = \sqrt{p} \cdot u' \tag{40}$$

The simplest non-trivial example that allows dual encoding is the rectangle that is 4 times as long as it is wide, $p = 4$. Then, we can find one count for the Width, n, with unit, u', and a second count for the length, $m = 2n$, with unit, $u'' = 2u'$. The reciprocal error in the first case, u'/n, and the reciprocal error in the second case, $u''/m = (2u')/(2n) = u'/n$, then being identical as required to fit the type of situation found in the room of the King's Chamber, where all the variances in the lengths are of the same order approximately $0.26''$. The room would then be described as being,

$$W \times L \;=\; (\frac{m}{4}) \cdot u'' \times m \cdot u'' \;=\; (\frac{2n}{4}) \cdot (2u') \times (2n) \cdot (2u') \;=\; n \cdot u' \times 4n \cdot u'.$$

the smallest such rectangle being, 1×4, which then has a total perimeter length of 10. In this example, one unit, u'', is simply double the other unit, $u'' = 2u'$, and the count of these units that make up a side is just double or half the other as the case may be. Now, we could pick any pair of numbers from the set of squares for our rectangle, say, $W \times L = 4h \times 9h$, and relax the requirement that the counts, n and m, be both whole numbers, accepting rational ratios in the analysis. That would allow for setting of a wider assortment of rectangles in the construction plan. But, this is outside the scope of the current discussion. Our King's Chamber is stuck with the 2 by 1 design, and that's what we have to deal with at the moment.

Analytically, we see that we can't get both, $n = 28$ and $m = 40$, if we also want to fix, $L = 2 \cdot W$, exactly, with common reciprocal error; but have already pointed out that somehow the builders of the Great Pyramid managed to seemingly circumvent this problem, by using 8 measurements, jiggling the vertices of the box around, to come up with a "best fit" to both, at least, a better fit than we'd expect from the data. So, now let us approach the problem in a slightly different way. We'll search for the best numbers, (n, m), that will fit Petrie's data, by considering just the East Wall and its parallel alone, and then just the North Wall and its parallel alone.

Another Approach. To do this, we construct an Error function, $E'(n, u)$, formed from the sum of the squares of the difference between the model and the data for the East Wall and its parallel, dividing the sum by 4, and taking the square root. Then, we differentiate this function with respect to the unit length, u, and set to zero, $\partial E'/\partial u = 0$, to find the particular value, $u = u'$, which minimizes this function; and then given a particular test value for the count, n, we compute, the unit, $u' = u'(n)$, and the error of fit, $E' = E'(n, u')$, at that point, and report the result in a table. Similarly, we construct an Error function, $E''(m, u)$, formed from the sum of squares using the North Wall data, differentiate and set to zero to find its unit length, $u = u''$, and calculate the corresponding values, (u'', E''), at particular count m. The formulas and the table of results for counts ranging from 10 to 60 appear below.

EAST WALL:

$$u' = \frac{n \cdot (206.30 + 206.43) + (n - \frac{1}{n}) \cdot (206.04 + 206.16)}{2(2n^2 - 2 + \frac{1}{n^2})} \tag{41}$$

$$E' = \surd\left(\frac{1}{4} \cdot ((n \cdot u - 206.30)^2 + (n \cdot u - 206.43)^2 + ((n - \frac{1}{n}) \cdot u - 206.04)^2 + ((n - \frac{1}{n}) \cdot u - 206.16)^2)\right) \tag{42}$$

NORTH WALL:

$$u'' = \frac{m \cdot (412.14 + 412.78) + (m - \frac{1}{m}).(411.88 + 412.53)}{2(2m^2 - 2 + \frac{1}{m^2})} \tag{43}$$

$$E'' = \surd\left(\frac{1}{4} \cdot ((m \cdot u - 412.14)^2 + (m \cdot u - 412.78)^2 + ((m - \frac{1}{m}) \cdot u - 411.88)^2 + ((m - \frac{1}{m}) \cdot u - 412.53)^2)\right) \tag{44}$$

The results indicate that the two counts, $n = 28$ and $m = 40$, are indeed the best fits to the model of reciprocal adjustment, for Width and Length, W and L, when the East Wall and opposite parallel are considered alone, and the North Wall and opposite parallel are considered alone. So, we effectively relax the strict requirement, $L = 2 \cdot W$, and common reciprocal error; allowing, $L \approx 2 \cdot W$, and differing reciprocal errors, instead. We can see the error of fit, E', decrease in size as we increase n until we hit, $n = 28$, after which it begins to increase again. So, the Width of the room is optimally divided into 28 equal parts; and we can see the other error of fit, E'', decrease until we hit, $m = 40$, and immediately start to increase, as we increase the count, m, so the Length of the room is optimally divided into 40 equal parts.

n	u'	E'	m	u''	E''	m	u_2''	E_2''
10	20.726428	0.905995	10	41.439470	1.971062	10	41.407060	1.940378
11	18.825929	0.725930	11	37.639711	1.615920	11	37.610273	1.579571
12	17.245753	0.589405	12	34.480385	1.348330	12	34.453418	1.305567
13	15.911003	0.483527	13	31.811752	1.142498	13	31.786872	1.092577
14	14.768495	0.399870	14	29.527477	0.981552	14	29.504383	0.923731
15	13.779406	0.332746	15	27.549938	0.854067	15	27.528391	0.787615
16	12.914722	0.278208	16	25.821130	0.752072	16	25.800935	0.676281
17	12.152325	0.233457	17	24.296829	0.669862	17	24.277826	0.584055
18	11.475054	0.196474	18	22.942723	0.603247	18	22.924779	0.506800
19	10.869385	0.165789	19	21.731778	0.549085	19	21.714780	0.441442
20	10.324523	0.140325	20	20.642404	0.504967	20	20.626259	0.385657
21	9.831737	0.119301	21	19.657150	0.469011	21	19.641775	0.337662
22	9.383895	0.102158	22	18.761754	0.439723	22	18.747080	0.296070
23	8.975111	0.088502	23	17.944450	0.415896	23	17.930415	0.259792
24	8.600485	0.078046	24	17.195439	0.396551	24	17.181990	0.227958
25	8.255904	0.070544	25	16.506499	0.380882	25	16.493588	0.199872
26	7.937890	0.065716	26	15.870676	0.368228	26	15.858263	0.174967
27	7.643483	0.063201	27	15.282052	0.358041	27	15.270099	0.152780
28	**7.370147**	**0.062556**	28	14.735555	0.349870	28	14.724030	0.132929
29	7.115696	0.063315	29	14.226818	0.343347	29	14.215691	0.115098
30	6.878238	0.065051	30	13.752055	0.338165	30	13.741299	0.099022
31	6.656125	0.067414	31	13.307971	0.334076	31	13.297562	0.084477
32	6.447915	0.070147	32	12.891684	0.330875	32	12.881601	0.071275
33	6.252341	0.073068	33	12.500663	0.328397	33	12.490886	0.059256
34	6.068287	0.076052	34	12.132674	0.326505	34	12.123184	0.048282
35	5.894764	0.079021	35	11.785739	0.325089	35	11.776521	0.038236
36	5.730892	0.081923	36	11.458101	0.324058	36	11.449139	0.029016
37	5.575888	0.084727	37	11.148193	0.323340	37	11.139474	0.020533
38	5.429051	0.087415	38	10.854614	0.322874	38	10.846124	0.012712
39	5.289752	0.089980	39	10.576105	0.322611	39	10.567833	0.005485
40	5.157425	0.092417	**40**	**10.311535**	**0.322513**	**40**	**10.303470**	**0.001207**
41	5.031558	0.094728	41	10.059883	0.322546	41	10.052014	0.007414
42	4.911690	0.096915	42	9.820224	0.322684	42	9.812543	0.013184
43	4.797402	0.098984	43	9.591722	0.322905	43	9.584220	0.018556
44	4.688314	0.100940	44	9.373614	0.323191	44	9.366283	0.023565
45	4.584077	0.102789	45	9.165208	0.323529	45	9.158039	0.028244
46	4.484375	0.104536	46	8.965869	0.323906	46	8.958856	0.032621
47	4.388919	0.106187	47	8.775018	0.324313	47	8.768155	0.036722
48	4.297443	0.107749	48	8.592125	0.324741	48	8.585405	0.040569
49	4.209704	0.109227	49	8.416702	0.325184	49	8.410119	0.044182
50	4.125475	0.110626	50	8.248300	0.325636	50	8.241849	0.047582
51	4.044552	0.111952	51	8.086506	0.326095	51	8.080181	0.050783
52	3.966743	0.113208	52	7.930938	0.326555	52	7.924735	0.053801
53	3.891872	0.114399	53	7.781244	0.327014	53	7.775158	0.056649
54	3.819776	0.115530	54	7.637097	0.327471	54	7.631123	0.059341
55	3.750302	0.116603	55	7.498194	0.327922	55	7.492329	0.061888
56	3.683311	0.117624	56	7.364255	0.328368	56	7.358495	0.064299
57	3.618671	0.118594	57	7.235017	0.328805	57	7.229358	0.066585
58	3.556262	0.119517	58	7.110238	0.329235	58	7.104677	0.068753
59	3.495969	0.120397	59	6.989691	0.329655	59	6.984224	0.070812
60	3.437686	0.121235	60	6.873163	0.330066	60	6.867787	0.072768

For the East Wall, $W = n \cdot u' = 28 \cdot 7.370''$, with a residual error on the order of, $\pm 0.06''$. In this case, the reciprocal error, $u'/n = 7.370''/28 = 0.26''$, is bigger than the size of the residual error, $0.26'' > 0.06''$ ✓, so that we can be confident that we have a good fit, and likely the correct model at work, so conclude the number, 28, does indeed evenly divide the Width of the room by the architect's intention. Notice that the error of fit, $0.062556''$, is very close to one-sixteenth of an inch, $1/16'' = 0.0625''$, so the error is on the order of a typical division on an imperial standard ruler that is usually calibrated in 8ths, 16ths, and 32nds, etc.. This result might lead us to think that it could very well be an artifact of the measuring tools used by the British archaeologist W.M. Flinders Petrie himself, in going about his measurements. However, Petrie reports that his measuring rods were graduated in 100ths of an inch, which would seem to invalidate this hypothesis; unless we presume he was perhaps subconsciously selecting the more common nearest 16ths when the 100ths graduation mark readings were found to be somewhat ambiguous during application of the measuring tool. Plugging in the fit parameters, $(n, u') = (28, 7.370'')$, back into the model to see how well they fit, we find all residual errors on the order of that very same $\frac{1}{16}$th of an inch;

East Wall Ceiling Width:	$W = n \cdot u'$	$= 28 \cdot 7.370''$	$= 206.36''$	$= 206.30'' + 0.06''$
East Wall Floor Width:	$W = n \cdot u'$	$= 28 \cdot 7.370''$	$= 206.36''$	$= 206.43'' - 0.07''$
West Wall Ceiling Width:	$W' = (n - 1/n) \cdot u'$	$= (28 - 1/28) \cdot 7.370''$	$= 206.10''$	$= 206.04'' + 0.06''$
West Wall Floor Width:	$W' = (n - 1/n) \cdot u'$	$= (28 - 1/28) \cdot 7.370''$	$= 206.10''$	$= 206.16'' - 0.06''$

For the North Wall, $L = m \cdot u'' = 40 \cdot 10.312''$, with a residual error on the order of, $\pm 0.32''$. In this case, however, the reciprocal error, $u''/m = 10.312''/40 = 0.26''$, is smaller than the size of the residual error, $0.26'' < 0.32''$ **x**, so that we don't have a really good fit here, and the model is suspect, so we can't conclude definitively that the number, 40, is intended to evenly divide the Length of the room. Despite the fact that the alternate count, $m = 40$, is the optimal fit to the North Wall data, it doesn't fit within the tolerance we'd expect to have great confidence in this parameter value. We have two options here. We could throw out this part of the model. Since it didn't fit our expectation, and just keep the, $n = 28$, parameter value. Or, we could recognise the difficulty in fitting two units to a room with dimension profile, 1×2, as discussed in the analytical section above, and give some weight to the fact that the builders nevertheless appeared to do better than would be expected in fitting both numbers in the construction, and include it just because of that observation. Plugging in the values, $(m, u'') = (40, 10.312'')$, into the North Wall model to see how well they fit, we find the residual errors are all now on the order of $\frac{1}{3}$rd of an inch;

North Wall Ceiling Length:	$L = m \cdot u''$	$= 40 \cdot 10.312''$	$= 412.48''$	$= 412.14'' + 0.34''$
North Wall Floor Length:	$L = m \cdot u''$	$= 40 \cdot 10.312''$	$= 412.48''$	$= 412.78'' - 0.30''$
South Wall Ceiling Length:	$L' = (m - 1/m) \cdot u''$	$= (40 - 1/40) \cdot 10.312''$	$= 412.22''$	$= 411.88'' + 0.34''$
South Wall Floor Length:	$L' = (m - 1/m) \cdot u''$	$= (40 - 1/40) \cdot 10.312''$	$= 412.22''$	$= 412.53'' - 0.31''$

Further investigation of the North Wall survey data suggests that there appears to be a problem with Petrie's data for the lengths of the room at the floor level. To test this thesis, we remove the floor data and run the analysis again with just the ceiling rectangle values, reconstructing our optimal unit to reflect just 2 data values, u_2'', and error function for these 2 data values, E_2''.

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NORTH WALL: - [revised, top only]
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$$u_2'' = \frac{m \cdot (412.14) + (m - \frac{1}{m}).(411.88)}{(2m^2 - 2 + \frac{1}{m^2})} \tag{45}$$

$$E_2'' = \surd\left(\frac{1}{2} \cdot ((m \cdot u - 412.14)^2 + ((m - \frac{1}{m}) \cdot u - 411.88)^2\right) \tag{46}$$

The final set of columns on the right of the table shows the re-run when restricting the least squares fit to the top of the chamber.

For the top North Wall, $L = m \cdot u_2'' = 40 \cdot 10.303''$, with a residual error on the order of, $\pm 0.0012''$. This is an excellent result, since the reciprocal error, $u_2''/m = 10.303''/40 = 0.26''$, is much larger than the size of the residual error, $0.26'' > 0.0012''$ ✓, so we can be confident that we have an excellent fit here, and our faith is restored in the working model. Hence the idea that the number, 40, divides the length of the room is once again a claim justified. The error of fit here, $\pm\, 0.0012''$, is quite small, and plugging in the fit parameters, (m, u_2''), back into the model yields;

North Wall Ceiling Length:	$L = m \cdot u_2''$	$= 40 \cdot 10.303''$	$= 412.12''$	$= 412.14'' - 0.02''$
South Wall Ceiling Length:	$L' = (m - 1/m) \cdot u_2''$	$= (40 - 1/40) \cdot 10.303''$	$= 412.86''$	$= 411.88'' - 0.02''$

Here, the small error results in an artifact of round-off error skewing the fit, appearing to show the same error, $-0.02''$, for both North and South wall lengths. However, if instead of truncating the unit to 3 decimals, $u_2'' = 10.303''$, we use the actual numerical value found, $u_2'' = 10.303470''$, we find the two sided fit as would be expected, $\pm\, 0.0012''$:

North Wall Ceiling Length:	$L = m \cdot u_2''$	$= 40 \cdot 10.303470''$	$= 412.1388''$	$= 412.14'' - 0.0012''$
South Wall Ceiling Length:	$L' = (m - 1/m) \cdot u_2''$	$= (40 - 1/40) \cdot 10.303470''$	$= 411.8812''$	$= 411.88'' + 0.0012''$

Petrie's Royal Cubit. Now we have used the measurements from the floor and the ceiling to compute the results above, and find the optimal counts and unit lengths. But, Petrie argues (p.28, 1990 ed) that we should only use the measurements on the floor level, for accurate determination of the unit length, because, he reasons, the foundations of the room would first be laid at the base of the walls, where everything would be most expected to be carefully set out, before the walls were put in place. According to his reasoning, we might expect the 4 measures around the perimeter of the floor, therefore, to give us the best values for analysis. To test this thesis out, we use our best parameters, $(n, u') = (28, 7.370'')$, and plug them into the model for the measures on the floor alone, to see how well they fit.

East Wall Floor Width:	$W = n \cdot u'$	$= 28 \cdot 7.370''$	$= 206.36''$	$= 206.43'' - 0.07''$
West Wall Floor Width:	$W' = (n - 1/n) \cdot u'$	$= (28 - 1/28) \cdot 7.370''$	$= 206.10''$	$= 206.16'' - 0.06''$
North Wall Floor Length:	$L = 2n \cdot u'$	$= 56 \cdot 7.370''$	$= 412.72''$	$= 412.78'' - 0.06''$
South Wall Floor Length:	$L' = (2n - 1/n) \cdot u'$	$= (56 - 1/28) \cdot 7.370''$	$= 412.46''$	$= 412.53'' - 0.07''$

The first thing we notice, immediately, is that although the residual errors are all at that same level of $\frac{1}{16}$th of an inch, this time, however, all the errors are on one side of the data. Our East Wall model produces consistently lower size lengths than the actual data Petrie measures for the room. This means we expect that we can adjust our unit of length and get an even better fit. So, we set up a similar error function, taking the squares of the difference between the model and data, as before, this time using the 4 floor dimensions, and differentiate the function, set to zero, to find the new and improved unit length, $u = u'''$, to fit this data when, $n = 28$.

$$u''' = \frac{28 \cdot 206.43'' + 56 \cdot 412.78'' + (28 - \frac{1}{28}) \cdot 206.16'' + (56 - \frac{1}{28}) \cdot 412.53''}{28^2 + 56^2 + (28 - \frac{1}{28})^2 + (56 - \frac{1}{28})^2} = 7.371428'' \tag{47}$$

East Wall Floor Width:	$W = n \cdot u'''$	$= 28 \cdot 7.371428''$	$= 206.40''$	$= 206.43'' - 0.03''$
West Wall Floor Width:	$W' = (n - 1/n) \cdot u'''$	$= (28 - 1/28) \cdot 7.371428''$	$= 206.14''$	$= 206.16'' - 0.02''$
North Wall Floor Length:	$L = 2n \cdot u'''$	$= 56 \cdot 7.371428''$	$= 412.80''$	$= 412.78'' + 0.02''$
South Wall Floor Length:	$L' = (2n - 1/n) \cdot u'''$	$= (56 - 1/28) \cdot 7.371428''$	$= 412.54''$	$= 412.53'' + 0.01''$

With the first optimization, using the East Wall, where our residual error is $0.06''$, the Royal Cubit is $28 \times 7.370''/10 = 20.636'' \pm 0.006''$; this is comparable to Petrie's Royal Cubit measure of $20.632'' \pm 0.004''$, as the ranges overlap. Now, here with the different fit by re-optimizing to just the floor measurements, following Petrie's direction to use just the floor values, we have a Royal Cubit measure of $28 \times 7.371428''/10 = 20.6399984'' = 20.640'' \pm 0.002''$, and the new length measure is higher and beyond the range of values in Petrie's measure.

Petrie's Royal Cubit can be obtained by simply adding the 4 floor measurements to get the perimeter of the room in inches, then dividing by 60 cubits, which is the nominal perimeter of the $10\,\mathrm{cu} \times 20\,\mathrm{cu}$ room,

$$\begin{aligned} \text{Petrie's Royal Cubit} &= \frac{(206.43'' + 412.78'' + 206.16'' + 412.53'') \pm 0.26''}{60 \text{ cubits}} \\ &= 20.631667'' \pm 0.004333'' \approx (20.632 \pm 0.004)\ \frac{\text{inches}}{\text{cubit}} \end{aligned} \tag{48}$$

It doesn't seem to occur to Petrie, that part of the variances found in the measurements of the room might be deliberate, so he simply averages over them, as if they are all just common measurement errors. When we apply our new model, which postulates that *the principle of reciprocal adjustment* was used by the architect, to account for part of the variances in the measures, we get a much closer fit to Petrie's own data, halving his implicated error of measure from $\pm.004''$ to just $\pm.002''$. This adds a certain measure of confidence to the new hypothesis. Of course, as with all models constructed to fit raw data, one always opens oneself to the usual criticism that with sufficient additional parameters, one can fit anything as closely as desired. However, note that in this particular instance, for *our* model, the additional terms involve the reciprocal, $1/n$, of the same count parameter inherent in Petrie's model, n, so we have not increased the number of independent free parameters in the new model.

Just as Petrie assumes the room was intended to be, $W \times L = n' \cdot u \times 2n' \cdot u = 10cu \times 20cu$, and so arrive at the divisor, $2 \cdot (n' + 2n') \cdot u = 60cu$, for perimeter, with his simple rectangle model, we say, $W = n \cdot u''', L = 2n \cdot u''', W' = (n - 1/n) \cdot u''', L' = (2n - 1/n) \cdot u'''$, allowing us to give some additional account of the actual observed variances in the measures. But, $n \cdot u''' = 28 \times 10d = 10 \times 28d = 10 \times 1cu = n' \cdot u$, so our, $n = \frac{28}{10} \cdot n'$, and, $u''' = \frac{10}{28} \cdot u$, and our free independent parameter, n, is proportional to Petrie's, n', so it is essentially the same one free count parameter. In both Petrie and our model, then, a nominal 1×2 rectangle is assumed, and a pair of '*count* and *unit*' parameters, '(n', u)' or '(n, u''')', fit to the raw data. Although formula complexity is increased, model complexity remains the same in terms of independent degrees of freedom used to describe the data. Incidentally, this is another good reason that would motivate an architect to use this particular reciprocal adjustment technique in design, because it doesn't require the introduction of any additional free degrees of freedom to the layout, and yet it is effective in encoding the metrology to convey the units of measure used in the plan.

Estimating tolerance ranges:

For setting and checking the lengths of the room, to confirm they are within theoretical tolerance specifications during construction, and they fit the reciprocal theory when survey measurements are made afterwards, we use the formula established in *The Reciprocal Theory* section above, now with the specific values for the counts, $n = 28$ and $n = 40$.

$$\frac{1}{2} \cdot \left(\frac{1}{40^2} + \frac{1}{41^2} \right) < \frac{\text{meas}\,\langle \Delta L(40) \rangle}{L} < \frac{1}{2} \cdot \left(\frac{1}{40^2} + \frac{1}{39^2} \right)$$

$$\frac{3281}{5379200} < \frac{\text{meas}\,\langle \Delta L(40) \rangle}{L} < \frac{3121}{4867200}$$

$$0.00060994200\cdots < \frac{\text{meas}\,\langle \Delta L(40) \rangle}{L} < 0.00064123110\cdots$$

$$\frac{1}{1639.500} < \frac{\text{meas}\,\langle \Delta L(40) \rangle}{L} < \frac{1}{1559.500}$$

$$\frac{1}{2} \cdot \left(\frac{1}{28^2} + \frac{1}{29^2} \right) < \frac{\text{meas}\,\langle \Delta W(28) \rangle}{W} < \frac{1}{2} \cdot \left(\frac{1}{28^2} + \frac{1}{27^2} \right)$$

$$\frac{1625}{1318688} < \frac{\text{meas}\,\langle \Delta W(28) \rangle}{W} < \frac{1513}{1143072}$$

$$0.00123228542\cdots < \frac{\text{meas}\,\langle \Delta W(28) \rangle}{W} < 0.00132362616\cdots$$

$$\frac{1}{811.500} < \frac{\text{meas}\,\langle \Delta W(28) \rangle}{W} < \frac{1}{755.500}$$

Ideally, we would like the blueprint value for at least the reference length, L, and reference width, W, but given that this is not available, we must estimate these values with the same survey data which is attempting to verify the fit.

So, we begin by exploring the two relevant ranges, for $\Delta L(40)$ and $\Delta W(28)$,

$$\frac{L}{1639.500} < \text{meas}\,\langle \Delta L(40) \rangle < \frac{L}{1559.500} \tag{49}$$

$$\frac{W}{811.500} < \text{meas}\,\langle \Delta W(28) \rangle < \frac{W}{755.500} \tag{50}$$

For the rectangle at the ceiling of the King's Chamber, Petrie's data tells us:

Top: $L = 412.14'', L' = 411.88'', W = 206.30'', W' = 206.04''$

$$\frac{412.14''}{1639.500} < \text{meas}\,\langle \Delta L(40) \rangle < \frac{412.14''}{1559.500} \tag{51}$$

$$0.251382'' < \text{meas}\,\langle \Delta L(40) \rangle < 0.264277'' \tag{52}$$

$$\text{meas}\,\langle \Delta L(40) \rangle = L - L' = 412.14'' - 411.88'' = 0.26'' \tag{53}$$

$$0.251382'' < 0.26'' < 0.264277'' \ \checkmark \tag{54}$$

$$\frac{206.30''}{811.500} < \text{meas}\,\langle \Delta W(28) \rangle < \frac{206.30''}{755.500} \tag{55}$$

$$0.254221'' < \text{meas}\,\langle \Delta W(28) \rangle < 0.273064'' \tag{56}$$

$$\text{meas}\,\langle \Delta W(28) \rangle = W - W' = 206.30'' - 206.04'' = 0.26'' \tag{57}$$

$$0.254221'' < 0.26'' < 0.273064'' \ \checkmark \tag{58}$$

For the rectangle at the floor of the King's Chamber, Petrie's data tells us:

Base: $L = 412.78'', L' = 412.53'', W = 206.43'', W' = 206.16''$

$$\frac{412.78''}{1639.500} < \text{meas}\,\langle \Delta L(40) \rangle < \frac{412.78''}{1559.500} \tag{59}$$

$$0.251772'' < \text{meas}\,\langle \Delta L(40) \rangle < 0.264687'' \tag{60}$$

$$\text{meas}\,\langle \Delta L(40) \rangle = L - L' = 412.78'' - 412.53'' = 0.25'' \tag{61}$$

$$0.251772'' \not< 0.25'' < 0.264687'' \quad \text{x} \tag{62}$$

$$\frac{206.43''}{811.500} < \text{meas}\,\langle \Delta W(28) \rangle < \frac{206.43''}{755.500} \tag{63}$$

$$0.254381'' < \text{meas}\,\langle \Delta W(28) \rangle < 0.273236'' \tag{64}$$

$$\text{meas}\,\langle \Delta W(28) \rangle = W - W' = 206.43'' - 206.16'' = 0.27'' \tag{65}$$

$$0.254381'' < 0.27'' < 0.273236'' \quad \checkmark \tag{66}$$

The error here, marked with a red x, shows that the measured difference, $0.25''$, falls outside the range required by the theoretical specifications. This is either due to Petrie's measurement error, or, if valid, a possible error in wall placement by the original builders; or, a result of the disturbance caused by earthquakes which shifted the blocks in the room, which, although recognised and compensated for by Petrie, he may have over compensated here for what he thought was the effect of the disturbance, resulting in his earthquake adjusted value falling outside its specification range. Petrie said he had to 'subtract' small amounts to account for the opening of the seams between wall blocks, and since the value here falls short of the range, it looks like he might have subtracted a bit too much here on the longer length, $L = 412.78''$, or not enough on the shorter length, $L' = 412.53''$, in adjusting his raw measurements. Given that all the other measured differences are within their specifications, as illustrated by the red checkmark ✓ above, the other calculations support the reciprocal theory well enough to question this one anomalous result. A mere additional $.002''$ would put it back in range, so given that Petrie only publishes his numbers to the nearest $0.01''$, it could also just be simply explained by round-off error.

For example;

$$\forall\, L \in \{L : 412.7750'' < L < 412.7849''\} \implies L \to 412.78'' \tag{67}$$

$$\forall\, L' \in \{L' : 412.5250'' < L' < 412.5349''\} \implies L' \to 412.53'' \tag{68}$$

$$\therefore \quad max(\Delta L(40)) = 412.7849'' - 412.5250'' = 0.2599'' \tag{69}$$

$$min(\Delta L(40)) = 412.7750'' - 412.5349'' = 0.2401'' \tag{70}$$

$$0.2401'' < \text{meas}\,\langle \Delta L(40) \rangle < 0.2599'' \tag{71}$$

and, any value in the sub-range:

$$0.251772'' < \text{meas}\,\langle \Delta L(40) \rangle < 0.2599'' \tag{72}$$

would satisfy the specs, but would be hidden by the reported precision.

It is not impossible that more of Petrie's data suffers from an unusual confluence of accidental multiple round-off errors, that conspire to favor the illusion that his data fits well with the Reciprocal Theory, whilst, in fact, we may be looking at an artifact caused by the precision limitations in his measuring tools. Although this seems unlikely, re-measuring the King's Chamber with an additional order of magnitude, with .001″ precision, instead of .01″, should settle any doubts. (See `APPENDIX A` *for discussion on the effect of round-off on Precision Errors.)*

Now, one advantage of building the walls of the King's Chamber in 5 courses of stone blocks, is that it gives the construction engineers an opportunity to correct for possible small errors during the placement. If, after placing the lowest course, and measuring back the dimensions, to confirm the fit, it is found that the values of the four lengths, $\{L, L', W, W'\}$, are off from specification, as seen in the above example, the higher courses can be placed with this in mind, using adjustments in their placements, to compensate for the initial error. Hence, we would expect any unintended variances in construction of the walls near the floor, to be compensated for and corrected by the walls' upper courses, so that some average of results from measuring of the floor rectangle and the ceiling rectangle would be expected to better yield the architect's design intent.

Note that this view is contrary to the opinion expressed by Petrie, who feels that the lowest course should have the most accurate placement and hence measures, since this is at the 'foundation' of the room, which is first placed, and would be expected to have attracted the greatest attention to detail and care in placement. Petrie feels that the measurements in upper courses would be expected to be less accurate than the base, not the reverse as we have expressed here. Petrie uses this argument to justify his use of the floor dimensions alone to establish the measure of the Royal Cubit.

Average of the Top Rectangle and the Base Rectangle Dimensions to compensate for possible wall placement error occuring during construction:

$$\text{meas}\,\langle \Delta L(40) \rangle :$$

$$\frac{0.251382'' + 0.251772''}{2} < \frac{0.26'' + 0.25''}{2} < \frac{0.264277'' + 0.264687''}{2} \quad (73)$$

$$0.251577'' < 0.255'' < 0.264482'' \quad \checkmark \quad (74)$$

$$\text{meas}\,\langle \Delta W(28) \rangle :$$

$$\frac{0.254211'' + 0.254381''}{2} < \frac{0.26'' + 0.27''}{2} < \frac{0.273064'' + 0.273236''}{2} \quad (75)$$

$$0.254296'' < 0.265'' < 0.273150'' \quad \checkmark \quad (76)$$

While Petrie's view is a valid "opinion," our view is consistent with the actual measures Petrie reports, at least, given our application of the "Reciprocal Theory." For it does seem that the ceiling measurements compensate for the apparent error found in the floor measurements, which problem is fixed when the calculation of their average is done. Given that, despite the best intentions, the builders moving these massively heavy blocks around, attempting to place them with such high level of precision and tolerance as required to encode the parameters, are apt to make some slight mistakes, we'd expect the multiple courses of stone in the walls to be a feature design in anticipation of such possible mistakes, by providing a mechanism for correcting them. This only adds weight to our view, that the average of upper and lower course rectangles may be intended to better reflect the design.

Calculating wall flatness tolerance requirement:

$$
\begin{array}{ll}
L = 412.14'', L' = 411.88'' & 0.264277'' - 0.251382'' = 4 \times 0.00322375'' \approx 4 \times 1''/310 \\
W = 206.30'', W' = 206.04'' & 0.273064'' - 0.254211'' = 4 \times 0.00471325'' \approx 4 \times 1''/212 \\
L = 412.78'', L' = 412.53'' & 0.264687'' - 0.251772'' = 4 \times 0.00322875'' \approx 4 \times 1''/310 \\
W = 206.43'', W' = 206.16'' & 0.273236'' - 0.254381'' = 4 \times 0.00471375'' \approx 4 \times 1''/212
\end{array}
$$

The first approach is to take the theoretical specification range for the measured values, meas $\langle \Delta L(40) \rangle$, and, meas $\langle \Delta W(28) \rangle$, divide this whole range by 2, to get the distance on either side of the mid-range, then divide that by 2 again, to allocate one half of this to each of the parallel walls. We end up with about $1''/300$ for the lengths, and about $1''/200$ for the widths; so that the East and West walls should be ground and polished to a flatness of about $1''/300$, and the North and South walls ground and polished to a flatness of about $1''/200$. From a purely construction point of view, this crude calculation gives us a working minimum for polishing the walls. However, as we have seen, the surveyor, in this case Petrie, comes afterwords and measures with some error of measurements of his own, so that if we are to account for that, we might want to improve these numbers as follows. Taking the average of ceiling and floor values and ranges determined before;

meas $\langle \Delta L(40) \rangle$:

$$0.251577'' < 0.255'' < 0.264482'' \quad \checkmark \tag{77}$$
$$0.255'' - 0.003423'' < 0.255'' < 0.255'' + 0.009482'' \tag{78}$$
$$\mathbf{min}[\,|-0.003423''|\,,\,|+0.009483''|\,] = 0.003423'' \tag{79}$$
$$\mathbf{tol} = 0.003423''/2 = 0.0017115'' = 1''/584.3 \approx 1''/600 \tag{80}$$

meas $\langle \Delta W(28) \rangle$:

$$0.254296'' < 0.265'' < 0.273150'' \quad \checkmark \tag{81}$$
$$0.265'' - 0.010704'' < 0.265'' < 0.265'' + 0.008150'' \tag{82}$$
$$\mathbf{min}[\,|-0.010704''|\,,\,|+0.008150''|\,] = 0.008150'' \tag{83}$$
$$\mathbf{tol} = 0.008150''/2 = 0.004075'' = 1''/245.4 \approx 1''/250 \tag{84}$$

Then notice, the difference, $\Delta L(n)$, does not naturally fall in the middle of the theoretical tolerance range. It is normally skewed to one side, so in determining the safe conditions that ensure the construction will fit the design specs, the conservative decision is to ensure that the smaller of the tolerance ranges on the two sides be set as the limit within which this delta value is to be established.

For the Lengths of the room, the difference of distance between the East and West walls, are best set to, $\Delta L(40) = 0.255'' \pm 0.003423''$, and for the Widths of the room, the difference between the North and South walls, are best set to, $\Delta W(28) = 0.265'' \pm 0.008150''$. Both the error of wall placement, and the error caused by the roughness of the surface of the walls, will be expected to contribute to that error value. IF we assume that the wall placement error has been removed by taking the average of the ceiling and floor results, then we can attribute the remaining error to the roughness of the walls. Since the pair of walls can both contribute to the error on the same side of the delta value, we must divide this error by 2 to attribute half of that error to each of the nominally parallel wall surfaces.

Hence the granite stone blocks on the East and West walls must be ground down and polished to a minimum of 1/600 of an inch, while the North and South walls must be ground and polished to a minimum of 1/250 of an inch. Overall then, the vertical walls should be polished to at least 1/600 of an inch, or better, to guarantee the measurements are within theoretical spec, and not otherwise adversely influenced by the roughness of the stone surfaces. Currently, there doesn't seem to be any survey data that reports the actual measure of flatness achieved by the builders in grounding and polishing the walls; so we are left with this theoretical calculation to estimate what the numbers could look like, as a technical explanation for 'why' the walls seem to have been polished.

Summary of the King's Chamber Results.

In summary, we find that by application of the *principle of reciprocal adjustment*, to the length and width data from Petrie's survey, that the King's Chamber is a room intended to reflect two counts, 28×56, and 20×40. The unit, u', in the first case being symbolically representative of a "pair of hands," or ten digits, $u' = 10d$; while the unit, u'', in the second case being symbolically representative of a "long span," or $1/2$ of a royal cubit, $u' = 14d = \frac{1}{2}\, cu$. Then, to simultaneously encode both of these systems in the same walls, by the pairing of these two systems together, we find this also then infers the smaller common unit of the "two-finger width," $u = 2d$, in which case the room count is also, 140×280. On recognising that unit, u, to be the width of a pair of fingers on the human hand, the count in the ultimate dimensions of the room is understood to be 280×560, on the scale with the unit as the digit, d.

The thesis here, is that the architect's intention is to focus on 'the counts,' for which exact and precise values are possible, as nearest integers to the calculated values. While, 'the units' are there to implicate the general type of thing that is being counted; in this case, 'hand pairs,' 'spans,' 'two-fingers,' 'digits,' and so on; things that would naturally have somewhat variable values in the natural world at any rate, but could nevertheless be recognised by similarity of the values uncovered with the things in the known world.

In other words, *that which can be precise, is made precise, and that which is naturally variable is left variable or inexact.*

The discovery of the "two-finger" unit in the walls, can be seen as a message to pay attention to the measurements, or to pay attention to that which is "precise," or "exact," in the measurements; that is to say, the actual "counts" found in the dimension of the room!

Of course, all of this is based on the premise that the Ancient Egyptians used this system of reciprocal adjustment in their metrology; and despite the apparent good fit of the theory to the survey data, there is one glaring problem with Petrie's floor data for the long side of the room. One might also claim that while Petrie's survey data precision, $0.01''$, might lead to these as indicative results, to be absolutely certain, the room should really be measured to one more decimal place, $0.001''$, to remove any question about round-off errors contributing to the appearance of fit (see APPENDIX A) that might not really exist.

4 The Socket Square

The Great Pyramid base lengths.

Having looked at the internal dimensions of the King's Chamber, which has some of the best measured and recorded data available in the whole Great Pyramid[2], let's turn our attention now to the external measures of this pyramid, examine the issues with the measurements available there, and see what sense we can make of the data that is currently available to us. Although several attempts were made, by notable people from John Greaves (1638-1640), the Napoleon French Savants (1798-1801), Col. Howard Vyse (1837), the British Ordinance Survey team Acton and Inglis (1865), Piazzi Smyth (1865), the English Royal Surveyors (1869) (who found the best measure up to their time giving the base length from socket to socket as, $9130.0''$), to measure the external dimensions of the pyramid before Petrie (1883), the rubble at the base was apparently not sufficiently cleared before his time to determine the measures with the best degree of precision. Petrie had to dig through the rubble in several spots, to get access to the features in order to make his measurements. He says,

> "*The casing of the Great Pyramid was found in situ on each side but only just in the middle of the faces ; the shafts were cut about 20 feet deep to reach it through the loose rubbish of chips... The pavement was found in many parts by cutting pits and trenches, and the edge of its rock bed was uncovered.*" (p.8 in 1990 ed.)

[2]Petrie says, "*The King's Chamber was more completely measured than any other part of the pyramid;*" (p.27 in 1990 ed; p.79 in 1883 ed); and in the new re-print of the abridged 1885 2nd edition, Dr. Zahi Hawass writes, "*Petrie's work on the Giza Plateau is still considered as an accurate and important archaeological investigation and a basic reference for the site*;" (p.98 in 1990ed).

Petrie gives the mean base side length[3], as $9068.8'' \pm .5''$, and height to apex of $5776.0'' \pm 7.0''$. (p.13 in 1990 ed; p.43 in 1883 ed). This amounts to $(9068.8 \pm .5)/(20.632 \pm .004) = 439.55cu \pm .11cu$, and, $(5776.0 \pm 7.0)/(20.632 \pm .004) = 279.95cu \pm .39cu$, using Petrie's own best measure for the cubit (p.28 in 1990 ed; p.81 in 1883 ed); or very nearly, $440cu$ and $280cu$, respectively, when rounded to the nearest cubit. This suggests we can sensibly claim that the *design* height to apex was intended to be 280 cubits exactly, as this number falls within the range of his probable error. But, as for the length, the cubit measure is too far from either whole number count on either side to be confident that either 440 or 439 should be the intended count of cubits there, as it seems necessarily to be of some intermediate value on the cubit scale; and this suggests we look elsewhere for an explanation of the base length measurements, if we seek an integral measure. As Petrie points out, the ancient Egyptian architects seemed to prefer designs with either integral lineal, or integral counts in the squares of the lineal measures (p.94, in 1990 ed; pp.221-222 in 1883 ed). So, we're always on the lookout for these integer quantities in the constructions. Here are Petrie's measurements for the lengths associated with the square base of the Great Pyramid, along with his commentary on probable errors of measure; we've also included our own rough sketch of the base squares to help illustrate what is being reported by Petrie;

The following are the dimensions of the Great Pyramid base:–

	Socket Sides	Base of Casing	Core-plane Sides
East	$9130.8''$	$9067.7''$	$8999.4''$
West	$9119.2''$	$9068.6''$	$9002.5''$
North	$9129.8''$	$9069.4''$	$9002.3''$
South	$9123.9''$	$9069.5''$	$9001.7''$
Mean	$9125.9''$	$9068.8''$	$9001.5''$
Mean diff.	$\pm 4.4''$	$\pm .65''$	$\pm 1.0''$

"And it must always be remembered that this very small mean error of the casing, .65 inch and 12", is that found through measurements of the rock sockets, and not that of the finally adjusted, and now destroyed, casing. This also includes errors of survey, which are probably ±.3 inch, and of settling the ancient points of casing and sockets which are about ±.5 inch. So that we can only say that the original base was probably more accurate than .65 inch in length and 12" in angle." - p.11 in 1990 ed; pp.38-40 in 1883 ed, "*The Pyramids and Temples of Gizeh*," W.M.Flinders Petrie.

We have taken the liberty to re-arrange the data taken from Petrie's table, to reflect our interest in the present analysis, and have left out the azimuth data, since that is of less immediate concern here. There are three nested squares that form the basis of the measurements of this pyramid base. The outermost square runs through the outermost corners of particular cut outs in the rock plateau below, referred to as the "*foundation sockets*," which form the corners of this square. The innermost square is where the core blocks of limestone are

[3]It is unclear how the previous ±.65 became ±.5 here, but we take it as given to illustrate the calculation. His error measures are the mean of the absolute differences, referred to as 'mean probable error'. Today, it's more common to use 'standard deviation' as a measure of reported error, which would be $\pm .72''$ here, when considering the 4 side lengths found to be sample data for the 'same' length.

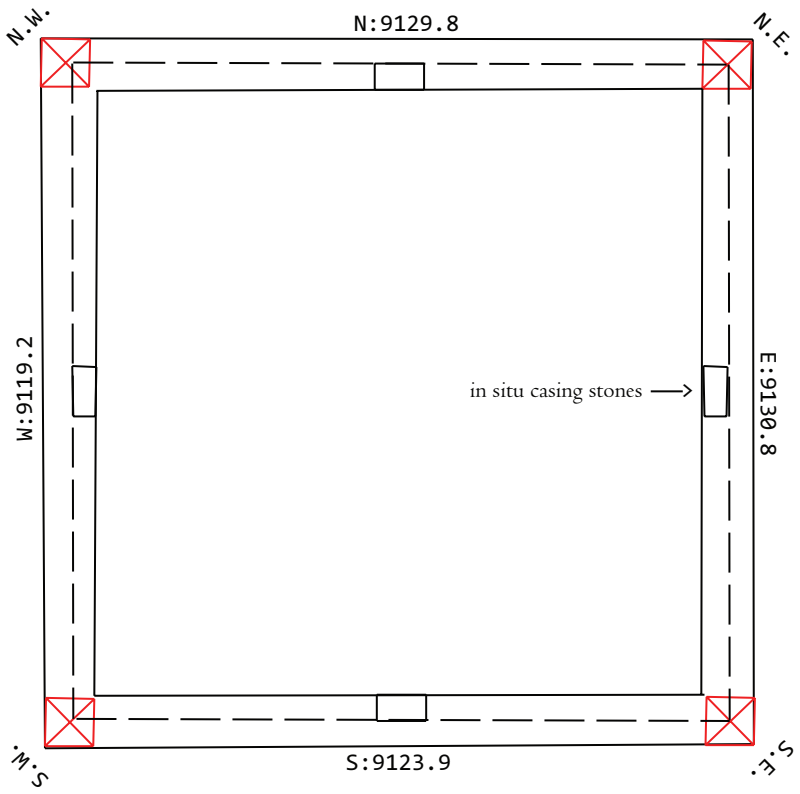

Figure 4: *The Great Pyramid Base Squares (not drawn to scale). Inner is the "core square " abt* $436\,cu \times 436\,cu$*, middle is the "casing square" abt* $440\,cu \times 440\,cu$*, outer is the "socket square" abt* $442\,cu \times 442\,cu$*, and the outermost "limit square" of* $448\,cu \times 448\,cu$ *not shown in the diagram; the red boxes in the corner are the locations of the four "foundation sockets" cut into the rock plateau.*

located, now stripped of almost all casing stones that once decked the surface of the Great Pyramid. Somewhere between the inner square and the outer square was the original square base of the pyramid, where the casing stones meet the pavement beneath the pyramid. That pavement itself is a raised platform of about 2 feet high over the rock plateau. When the rubble was cleared, a few casing stones were found still in place, *in situ* near the middle of each side, and from these stones an estimate of where the exact footprint of the original complete pyramid was originally located can be reconstructed. This square is shown in dashed lines in our drawing. Petrie assumes it should be a perfect square, and fits the best square he can to his data. He uses a number of triangulation measurements from numerous select points around the pyramid to "infer" and reconstruct the lengths and orientation angles of this square. While the outer *socket square* can be more or less measured directly, in principle, since

these rock cut-outs are still present[4], the *core square* and the *casing square* both require some degree of applied hypothesis model fit to the data obtained from triangulation procedures, and so must be obtained indirectly.

Here is where the main problem with the measures of the pyramid base occurs. What geometric model should we try "to fit" to the data? Is it a square, or a more general four sided polygon? (*The Great Pyramid actually has eight sides, as each face on the core is indented towards the center, with a very noticeable groove at the middle of each face, but Petrie examined the casing blocks in situ in the middle at the base level and found no indentation in the casing outer face itself, the casing there being considerably thicker taking up the slack at the back near the core, thus suggesting that the indentation is a feature of the core only, and not of the completed pyramid when it was originally covered with the white limestone casing blocks. pp.43-44 in 1883 ed*). As we have seen, incorporating the *principle of reciprocal adjustment* would require deliberate deviations from a perfect square. It should look like a square to implicate that geometric form, but should measure out to a more general four sided polygon on closer examination when we come to measure. As early as 1664, at least one visitor to Egypt claimed to have noticed that none of the Pyramids of Egypt have square bases. In his '*Life and Work*' VOL.III, Charles Piazzi Smyth writes the following;

> ...every one can see that the base of the Great Pyramid has four sides ; that the angles at the corners cannot be bery far from right angles ; and that the sides cannot be very unequal in length : but some men have hesitated to admit any high degree of exactness in those particulars, in face of the following passage from F. Vausleb, A.D. 1664,
>
> *'I have taken notice that none of the Pyramids' (bases) are alike, or perfectly square ; but that all have two sides longer than the others...'*
>
> – p.10, PROPOSITION I. 'THE BASE OF THE GREAT PYRAMID IS SQUARE?', '*Life and Work At the Great Pyramid. During the Months of January, February, March, and April,* A.D. *1865; With A Discussion of the Facts Ascertained*', By C. PIAZZI SMYTH, F.R.SS.L&E., In Three Volumes, VOL.III. 1867.

So, given that the outer socket square should have had the least problem being measured by Petrie in 1881-1883, we'll consider that first.

[4]Although, according to Glen Dash, the condition of these sockets have deteriorated since Petrie, making the 1883 measures the best available now. source: see his Footnote [1]: archive.org waybackmachine March 29, 2015 : https://web.archive.org/web/20150329123206/http://glendash.com/blog/2014/12/03/the-great-pyramid-diagonals-do-they-point-to-a-hidden-inner-platform-within-the-pyramid/

Let's apply the reciprocal adjustment to interpret the deviations on this Pyramid's Base Square. The first thing we notice, is that, both the King's Chamber and the Pyramid's Socket Square have East longer than West, and North longer than South. So, we compute the estimate of the number of divisions, n, for these parallels as before;

$$\text{East/West:} \quad n \quad = \quad 1/\sqrt{1 - 9119.2''/9130.8''} \quad = \quad 28.056 \quad \rightarrow \quad 28 \tag{85}$$

$$\text{North/South:} \quad n \quad = \quad 1/\sqrt{1 - 9123.9''/9129.8''} \quad = \quad 39.337 \quad \rightarrow \quad 39 \ ? \tag{86}$$

Perhaps, not so surprising, again we have the East/West dimension being divided up into 28 parts, just like in the King's Chamber. Clearly, and unambiguously, both the King's Chamber and the Pyramid Socket Square use this popular integer, 28, the same number of divisions on the Standard Royal Cubit Rod, that appears again in the Grand Gallery among the counts of the rectangular holes (or 'slots') in the side benches (or 'ramps'). So, we're confident that we're on the right track. However, the North/South dimension calculation is suggesting the count, $n = 39$. In the King's Chamber, these dimensions gave us, $n = 40$. But there, we had twice as many data points to contribute to the calculation. So, considering that everything else seems to line up, between the internal and external dimensions here, this makes us think that this, $n = 39$, result, should really be, $n = 40$. Lets see what would happen if we required, $n = 40$, to show up in the socket square North/South parallels.

$$\text{say,} \quad 40 \quad = \quad 1/\sqrt{1 - S'/9129.8''} \tag{87}$$

$$\therefore \quad S' \quad = \quad (1 - 1/1600) \cdot 9129.8'' \quad = \quad 9124.1'' \tag{88}$$

$$\delta S' \quad = \quad 9124.1'' - 9123.9'' \quad = \quad 0.20'' \quad < \quad |\pm .3''| \tag{89}$$

$$\text{or say,} \quad 40 \quad = \quad 1/\sqrt{1 - 9123.9''/S} \tag{90}$$

$$\therefore \quad S \quad = \quad 9123.9''/(1 - 1/1600) \quad = \quad 9129.6'' \tag{91}$$

$$\delta S \quad = \quad 9129.8'' - 9129.6'' \quad = \quad 0.20'' \quad < \quad |\pm .3''| \tag{92}$$

What we see, is that if Petrie's data for the South Side, S', was just a little longer and found to be, $S' = 9124.1''$, instead of his reported, $S' = 9123.9''$, then the count would indeed be that, $n = 40$, value, which we would expect. Or, if his North Side data point, S, was just a little shorter, $S = 9129.6''$, instead of, $S = 9129.8''$, again we'd get the anticipated, $n = 40$. Either way, this is just a $.2''$ difference, which amounts to, $.2'' \times 25.4\, mm/in \approx 5\, mm$. Hence, just a mere $5\, mm$ error in Petrie's measurements, for surveyed lengths over a distance of $760\, ft$, would shift the numbers to us give that, $n = 40$, result. Petrie himself, tells us that his survey of lineal lengths at the Giza plateau is subject to an estimated probable error of $\pm.3''$. And a shift of $.2''$ is well within this error range.

He also anticipates another error of an additional $\pm.5''$ might be considered due to the settling of the rock, blocks, and casing stones, over the millennia,

but it's not clear exactly when and where this additional $\pm.5''$ error should be considered, nor how he arrived at that error estimate. We'd expect Petrie to be completely familiar with his measuring instruments, and his own limitations in carrying out measurements with them, so we immediately accept his $\pm.3''$ error estimate, which is a function of applying these instruments to the work. And in the King's Chamber, Petrie gives a very clear and detailed explanation of how he made adjustments to his measures due to the movements of the blocks of rock caused by earthquakes opening up the joints, all displacements of which he could measure and account for the changes in dimensions over time. But, here in his survey of the external features, he gives no similar explanation of evidence of rock movements he observed and measured, to come up with this particular $\pm.5''$ figure, which makes it seem to be just a wild guess. However, we don't need that extra error adjustment to account for the thesis that, $n = 40$, is the result that should be present here. From the obvious correspondences in metrological motif found in both the internal King's Chamber and the external socket square, the evidence strongly suggests that the North/South count should be, $n = 40$, and given this is consistent with the $\pm.3''$ error reported, we'll adopt this value for the count—confirmation will follow below in the *subterranean chamber*.

$$S \times S = 28s' \times 28s' \tag{93}$$
$$S \times S = 40s'' \times 40s'' \tag{94}$$

$$28 = 2 \cdot 2 \cdot 7 \tag{95}$$
$$40 = 2 \cdot 2 \cdot 2 \cdot 5 \tag{96}$$
$$280 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \qquad \textbf{LCM(40,28) = 280} \tag{97}$$

$$S \times S = 280s \times 280s \tag{98}$$
$$= 28 \cdot (10s) \times 28 \cdot (10s) \implies s' = 10s \tag{99}$$
$$= 40 \cdot (7s) \times 40 \cdot (7s) \implies s'' = 7s \tag{100}$$

$$\text{so, roughly:} \quad s = \frac{9130.8''_{\text{E}} + 9119.2''_{\text{W}} + 9129.8''_{\text{N}} + 9123.9''_{\text{S}}}{4 \times 280} = 32.593'' \tag{101}$$

$$\text{or, more precisely:} \quad s_\circ = \frac{9130.8''_{\text{E}} + 9119.2''_{\text{W}} + 9129.8''_{\text{N}} + 9123.9''_{\text{S}}}{4 \times 280 - 10/28 - 7/40} = 32.608'' \tag{102}$$

This means that, like in the analysis of the internal King's Chamber dimensions, the external rectangular geometric figure here has two different units of measure being implicated in the dimensions. The East/West nominal parallels suggest a square, $S \times S = 28s' \times 28s'$, where, s', is the unit of measure; and at the same time the North/South nominal parallels suggest that same square is, $S \times S = 40s'' \times 40s''$. Now, in the King's Chamber, we discussed the difficulty encoding two different units of measure in a rectangle, $W \times L = w \times pw$, especially when that length to width ratio parameter, p, was not itself a perfect square. That meant, for the oblong rectangle, with, $p = 2$, we had to fight with the constraint limitation, $28 \times \sqrt{2} = 39.60 \neq 40$, and, $40/\sqrt{2} = 28.28 \neq 28$.

We showed how the architect got around this problem by using several additional pairs of parallels, to contribute to the calculation, which are available in a three-dimensional box with its 12 optional parallels. By picking 8 lines, polishing the surface of the walls in the vertical to guarantee these 8 lines could accurately encode the objective, and jiggling the vertices of the box around, the builder was able to overcome this limitation. But, here, with, $p = 1$, there's no *inherent* problem introducing two different units in the dimensions, and the two pairs of parallel lengths are quite sufficient. We easily encode 2 units by tweaking the square. Now, the least common multiple of the two numbers, 28 and 40, is 280, and thus for both of these units, s' and s'', to be simultaneously present in the side of the square, there must be another unit, smaller than both, so that, $S \times S = 280s \times 280s$, with, $s' = 10s$, and, $s'' = 7s$. A simple calculation reveals the unit, is roughly, $s = 32.593''$, in which case the other two larger units are found to be, $s' = 10s = 325.93''$, and, $s'' = 7s = 228.15''$; or with greater precision, that unit is, $s_\circ = 32.608''$, resulting in the larger units, $s' = 10s_\circ = 326.08''$, and, $s'' = 7s_\circ = 228.26''$.

Notice that it doesn't matter whether we use Petrie's data, $9123.9''$, or the suspected adjusted value, $9124.1''$, when computing this unit, $s = 32.593''$. That's because the error, $.2''$, only contributes, $.2''/(4 \times 280) = .0002''$, to the value of the unit calculation, and its effect is beyond the 3rd decimal place. We get the same number, $s = 32.593''$, or, $s_\circ = 32.608''$, with or without the adjustment. So, we'll continue to use Petrie's data as given in his text for simplicity.

Let's take a look at how well this model of reciprocal adjustment fits the data;

E	=	$28 \cdot 10s = 280 \times 32.593''$	=	$9126.04''$	=	$9130.8''$	$- 4.76''$
W	=	$(28 - 1/28) \cdot 10s = (280 - 10/28) \times 32.593''$	=	$9114.40''$	=	$9119.2''$	$- 4.80''$
N	=	$40 \cdot 7s = 280 \times 32.593''$	=	$9126.04''$	=	$9129.8''$	$- 3.76''$
S	=	$(40 - 1/40) \cdot 7s = (280 - 7/40) \times 32.593''$	=	$9120.34''$	=	$9123.9''$	$- 3.56''$
			avg. abs. err.		=		$(4.22'')$

E	=	$28 \cdot 10s_\circ = 280 \times 32.608''$	=	$9130.24''$	=	$9130.8''$	$- 0.56''$
W	=	$(28 - 1/28) \cdot 10s_\circ = (280 - 10/28) \times 32.608''$	=	$9118.59''$	=	$9119.2''$	$- 0.61''$
N	=	$40 \cdot 7s_\circ = 280 \times 32.608''$	=	$9130.24''$	=	$9129.8''$	$+ 0.44''$
S	=	$(40 - 1/40) \cdot 7s_\circ = (280 - 7/40) \times 32.608''$	=	$9124.53''$	=	$9123.9''$	$+ 0.63''$
			avg. abs. err.		=		$(0.56'')$

The rough calculation, s, yields a value that is on average $\pm 4\frac{1}{4}''$ away from the true value of the length of the sides (actually, it produces results consistently lower on average by about four and a quarter inch; so errors are always on one side, and when used in *this reciprocal model* it's clearly not such a fantastic fit! However, it's in line with the reported error, $\pm 4.4''$, which Petrie gives in his table above.); while, the precision result, $s_\circ$, is much closer, producing lengths differing only by about $\pm\frac{1}{2}''$ from the true values. This is such a great fit, that one might wonder why bother with the rough calculation result at all. However, first observe that these two values, s and $s_\circ$, are very near to

each other, and both round to the same value, $32.6''$, in situations when only one decimal place is required. Then, the important consideration of note here is the *thesis* that the builder introduces these variances in the lengths in order to *implicate* the division of the sides. So, we see the East is divided by 28, and the North is divided by 40, and it's really supposed to be a *square* form, with these divisions, so the '*idea*' being communicated is that this is a square with dimensions, $S \times S = 280s \times 280s$, which has perimeter, $P = 4 \times 280s$. If we use the precision result, say, $P = 4 \times 280s_\circ$, this will produce a path length about the four sided polygon that is longer than the actual perimeter! To use, $s_\circ$, properly, we must use the path formula that was employed to construct it, then, $P = [28 \times 10 + (28 - 1/28) \times 10 + 40 \times 7 + (40 - 1/40) \times 7] \times s_\circ$, will give the right path length around the perimeter. So, now we have to decide on the *context*, and use that as a guide to pick which value for the unit we will use, s or $s_\circ$, to interpret the design going forward. We will often use the rough result, drop the last decimal for simplicity, taking this unit as, $s = 32.59''$, as this will be consistent with the *context* of our application of the unit in interpreting the meaning of the Pyramid's design. One can always return to these results later and compare the changes in interpretation that may occur if the more precise unit was used.

The 17.5 inch West Displacement. Let us consider the discrepancies in the path length about this square, using the rough and precise units. We notice that the path difference between, $4 \times 280s_\circ = 36520.96''$, and the actual sum of the lengths, $9130.8''\text{E} + 9129.8''\text{N} + 9119.2''\text{W} + 9123.9''\text{S} = 36503.7''$, is, $36520.96'' - 36503.7'' = 17.26''$.

This is almost the length of the 24-digit 'common cubit,' $20.632'' \times 24/28 = 17.68''$. So the difference is significant. While, on the same path, $4 \times 280s = 36,504.16''$, which differs from the true path length by, $36,504.16'' - 36503.7'' = 0.46''$. This is only about a $\pm\frac{1}{2}''$ error; which is why we elect use it in such a *context*. But, we might rightfully ask why this error is not '*zero,*' since the *definition* of the rough calc unit, s, is given by, $s = P/(4 \times 280)$, and we're just reversing this operation. However, we rounded the value of s to 3 decimal places, and the *rounding up* required to arrive at the $32.593''$ is what makes the calculated path here just a bit longer, by 0.46 inch, than the true path length. So, what we see is that the number of significant figures used by Petrie's raw data, and by our subsequent rounding, limits the precision with which we can expect to know, and to reproduce, the true path length. We see that the 3 decimal place restriction can contribute as high an error of about $\pm\frac{1}{2}''$ to such calculation. This round-off error, of course, is greatly amplified by the nature of the particular calculation here, that finds a smaller length, s or $s_\circ$, reducing the precision to match the significant figures of the input data, then taking that lower precision version of the smaller length and multiplying by a relatively large number, 4×280, to recover the corresponding long length once again. The round-off error *accumulates* through the many effective additions inherent in such multiplication.

We expect Petrie encountered a similar effect, when he went about applying his triangulation and computation to find the *implicated* lengths on the Giza plateau in an analysis of his own raw data. However, Petrie says this sort of error only amounts to $\pm.3''$ on the surveyed lengths at Giza. This is probably due to the fact that he measured relatively long lengths, then turned them into other long lengths via triangulation, thus using fewer arithmetic operations than our small length to long length calculation. If instead of the rounded precision value, $s_\circ = 32.608''$, which is a *rounded down* figure, we use the higher precision 34-significant figures from our *Swiss Micros* DM42 High Precision Calculator, originally computed in the defining expression, then we find the discrepancy in path lengths is, $17.26'' + 0.09'' = 17.35''$. This, together with Petrie's own $\pm.3''$ error of survey, makes the actual discrepancy likely fall somewhere in the range: $17.35'' \pm .3''$ [5].

This immediately raises the question of whether there is a connection here to the well known $17.5''$ *west displacement* in the S.W. corner socket.

$$4 \times 280 \times 32.593'' \quad - 9130.8''_{\text{E}} - 9129.8''_{\text{N}} - 9119.2''_{\text{W}} \quad = \quad 9124.36''_{\text{S}} \quad = \quad 9123.9''_{\text{S}} + 0.46'' \tag{103}$$

$$4 \times 280 \times 32.608'' \quad - 9130.8''_{\text{E}} - 9129.8''_{\text{N}} - 9119.2''_{\text{W}} \quad = \quad 9141.16''_{\text{S}} \quad = \quad 9141.4''_{\text{S}} - 0.24'' \tag{104}$$

$$\text{The 'two' S.E. socket to S.W. socket line lengths:} \quad 9123.9'' + 17.5'' = 9141.4'' \tag{105}$$

Now, the socket square has one apparent ambiguity, which has caused large discrepancies in the reported results from different surveys in the past. While the S.E., N.E., and N.W. sockets, each have a single unequivocal point to establish the corners, at least as agreed upon by all surveyors to the Giza plateau, which amounts to the outermost corner of each socket that is farthest from the center of the pyramid, the S.W. socket has 'two' points that have seemed natural to surveyors as the defining points that mark the end of the line. This can best be understood by looking at the diagram from Davidson's Plate XX shown below. For example, the Royal Engineers (1874) survey gives the length from the S.E. corner of the S.E. corner socket, to the S.W. corner of the S.W. corner socket, marked by point X in the diagram, reporting this length as, S = $9140''$, while Petrie's 1881 survey reports that same length on the South Side of the Pyramid only up to the 'scored line' point marked with Z in the diagram.

[5] Properly, each of Petrie's socket square length values should be allotted an $\pm.3''$ size err, the path, $9130.8''_{\text{E}} \pm .3'' + 9129.8''_{\text{N}} \pm .3'' + 9119.2''_{\text{W}} \pm .3'' + 9123.9''_{\text{S}} \pm .3'' = 36503.7'' \pm (4) \times .3''$, then has up to 4 such possible adjustments, $4 \times (\pm.3'') = \pm 1.2''$, and we could write, $17.35 \pm 1.2''$, but some of these independent errors probably offset each other, it being equally likely that a $+.3''$ or a $-.3''$ occurs, so that the net error that results is smaller, and so we suggest here that at least one $\pm.3''$ remain for the whole path length, which is then sufficient to explain how the range could possibly contain the $17.5''$ displacement.

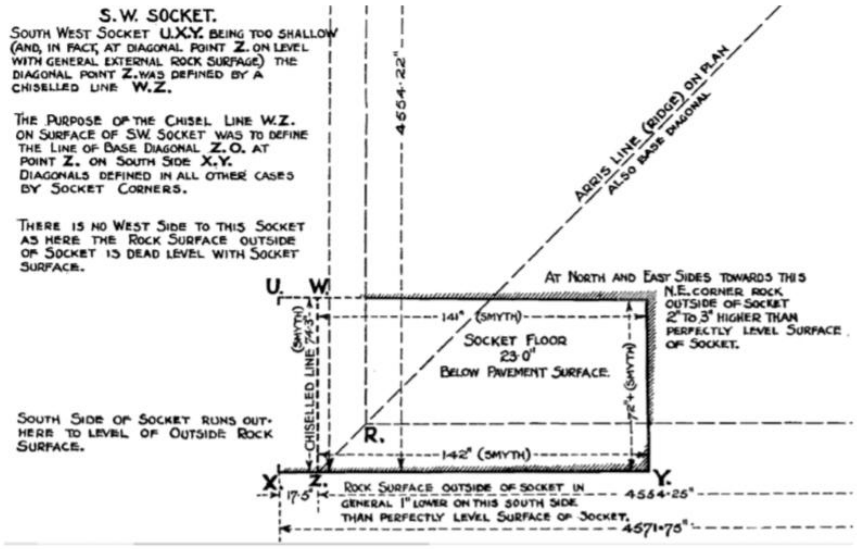

Figure 5: *Davidson's Plate XX - South West Socket*

As the diagram shows, the point X is farther west than Z by 17.5″. Hence, Petrie subtracts this 17.5″ displacement from the R.E. to find their equivalent length, S = 9140.0″ − 17.5″ = 9122.5″, to compare with his own figure, S = 9123.9″. In a footnote, remarking on the R.E. 1874 survey result, Petrie explains;

> "This is stated as 9,140; but as the other sides agree so closely with the 1874 and 1881 measures, this would seem to have been measured from a different point. Now the outer W. edge of the socket-block at the S.W. is 17.5 beyond the drawn line (which really defines the socket), and it is therefore about 9,140 from the S.E. corner ; hence this was probably taken as the corner, and 17.5 must therefore be deducted from the R.E. measure of this side in comparing it with the other surveys." (Source: footnote on p.206, Petrie's 1883 ed.)

Petrie's 1881 survey is considered to be the most accurate of all measurement results for the Giza plateau up to his time. So if we add this 17.5″ westward displacement to his more accurate South length, S = 9123.9″, we can determine what the 1881 survey would report for that R.E. south length to the actual end of the S.W. socket, which then becomes, S = 9141.4″.

Here is where two interesting observations can be made. The first is that these two lengths, S = 9123.9″ and S = 9141.4″, seem to be describing the difference between the two paths around the Great Pyramid socket square determined by the two, rough and precise, units, $s = 32.593''$ and $s_\circ = 32.608''$; one leading to point Z on Davidson's Plate XX, and the other to point X. The

second is that the South side, S = 9141.4″, is now longer than the North side, N = 9129.8″, the *reverse* of the situation met before, and when we examine the reason for the variance using the usual *principle of reciprocal adjustment* we get;

$$\text{South/North:} \quad n \quad = \quad 1/\sqrt{1 - 9129.8''/9141.4''} \quad = \quad 28.072 \quad \rightarrow \quad 28 \ ! \qquad (106)$$

Surprise, we get that number, 28, once again, as the division of the side length. This suggests the longer length, S = 9141.4″, was certainly intentional. Yet, the 'scored line' that Petrie uses for his end-of-line position suggests this was probably some sort of an error that had to be corrected afterwords. We can imagine the events unfolded something like this: The workmen cut the four corner sockets, after measuring, such that the South/North parallels would implicate the division, 28, for these sides. Then, the foreman supervising the work came and checked their work, finding they encoded the number, 28, in the wrong parallels, it should be only in the East/West lines, not the North/South, and besides they put the longer length on the wrong side, the South, instead of the North. The workmen probably complained, saying, "it's a square," and "What does it matter?"

Well, it does matter, because the plans describe a Grand Gallery to be built with 28 mortices along each of the ramps, and the orientation of that passage is along the North to South direction, which is parallel to the socket square East/West sides, and again the King's Chamber East/West walls on the plans are to be divided into 28 also, so the socket square must have the East/West parallels divided into 28 to be in conformity with the rest of the structure. The *repeating* of a theme, like many instances of the number, 28, and the orientation in which we find it, helps to resolve ambiguities that might arise in later measuring and interpreting the structure, as "*repetition*" is one way to reinforce an idea as being intentional and not simply accidental in a design plan. Therefore, East/West must be divided into 28, and if the South/North is also divided into 28, how will it be possible then to also encode that other division, 40, into the square? After explaining this to the workmen, the workers understood and re-measured the distances once again, and then "*chiseled*" a line inside the previously created S.W. socket rectangular rock cutout, as shown in Davidson's Plate XX diagram, which just happened to be 17.5″ from its west end; and this is the line that Petrie's says therefore marks the true end of the socket square, despite the fact that it's all the way inside the actual S.W. socket rectangle rock cutout.

Now, of course, while all of this might seem reasonable, there's still the puzzle of "why" then does it seem that the two south lengths, S = 9123.9″ and S = 9141.4″, appear to correspond *perfectly* to the two, rough and precise, units, $s = 32.593''$ and $s_\circ = 32.608''$, as if the builders are trying to suggest the connection was known, understood, and deliberately marked out in their plans all along, by this device of the ambiguous S.W. socket points? Of course, all this marking activity is going on *under* the eventual raised platform that today surveyors call the *pavement.* So, for these features to be visible to the inspect-

ing observers, after completion of the Great Pyramid, we expect there was some way of reflecting these two marks up on the top surface of that pavement, once the pavement blocks were put in place. Then the visitors to the Great Pyramid could view and contemplate the puzzle.

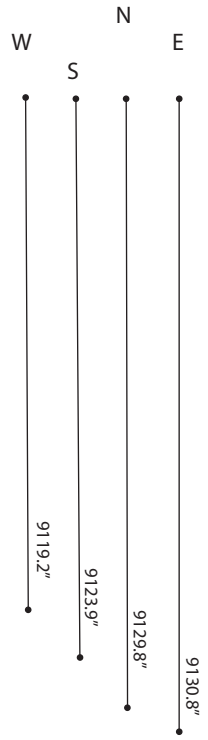

Figure 6: *Ordering of The Great Pyramid Socket Square Side Lengths (not drawn to scale).*

> *"Now when he had made an end of measuring the inner house, he brought me forth toward the gate whose prospect is toward the east, and measured it round about. He measured the* **EAST** *side with the measuring reed, five hundred reeds, with the measuring reed round about. He measured the* **NORTH** *side, five hundred reeds, with the measuring reed round about. He measured the* **SOUTH** *side, five hundred reeds, with the measuring reed. He turned about to the* **WEST** *side, and measured five hundred reeds with the measuring reed. He measured it by the four sides: it had a wall round about, five hundred reeds long, and five hundred broad, to make a separation between the sanctuary and the profane place."* (KJV, Ezekiel 42:15-20)

The Great Pyramid Socket Square Side Lengths follow the exact order in the "*attention to measurements*" as described in the Book of Ezekiel, demonstrated by the Angel, **E** → **N** → **S** → **W**. This appears to have been some kind of "*convention*" in the ordering of the cardinal points of the compass, probably related to the path of the sun in the sky. The sun rises in the East, and sets in the West. So, East is naturally before West. Then, the sun at its zenith moves North to South during the year, while performing its daily East to West routine. The combining of these two repeating movements gives rise to the ordering, as the Sun ends the combined cycle at the Winter Solstice on December 21st, at which point it is at its most Southern point in the sky (shortest day, and longest night; i.e as seen from Giza in the Northern

Hemisphere) when the sun sets in the West. Hence, South and West mark the end, leaving the North to follow immediately after the rising Sun in the East. This would explain the order, and the builders of the GP seem to follow this "sacred" order in their construction when they "tweaked" the square to encode the metrology, as the East Side (9130.8") is the longest length, and the natural starting point, followed by the North Side (9129.8") almost as long as the East, followed again by the South Side (9123.9"), and finally the shortest length is seen in the West Side (9119.2")– corresponding to the end of the 'shortest day' of the year. Notice that the setting sun in the South and West marks the 'end' of the combined cycle in this convention. The Jewish Calendar is in harmony with this pattern, as the day ends at sunset, and the new day begins as the next evening begins, as prescribed in the Book of Genesis 1:5 (KJV);

"And God called the light Day, and the darkness he called Night. And the evening and the morning were the first day."

Today, in most of the world, the Calendar day starts at midnight, and sunrise is the start of most people's actual day, obscuring the ancient convention of the setting sun as marking the end point. For the old Egyptian and Hebrew convention, a day is an evening first, followed by a morning next, and ends when the sun is in the lowest position in the sky once again at sunset. In this case, the end of the day and the end of the year are both together marked by the same visual sign, that of the sun being "seen" to reach its lowest point in the sky. So, for the ancient engineers, looking at the Great Pyramid, all these things would be immediately obvious. But for the modern engineer, it takes a bit of thought and reflection, to grapple with the conventions found in the builder's grand design, to ultimately understand the plan.

In considering the assignment of sides of a monument to *reference* lengths vs. their *apposites*, we turn to the documented remarks found in scripture for guidance from the way ancients thought about these things back in old times. We find the angel telling Ezekiel to "pay attention" and "record everything he sees the angel do." Then the angel proceeds to perform a "*measuring ritual*" about the temple and temple grounds, which reveals the importance attributed to the "order" of arrangements in the way things are done. This tells us the order of importance for the measures is, East-North-South-West, and the Great Pyramid corroborates this ordering in the assignments of lengths of the sides of the socket square. Since the socket square is obviously the "reference square," as surveyors have pointed out its diagonals set and establishes the orientation of the pyramid, we see that the assignments are then naturally, W, W', for East/West, and, L, L' for North/South. Of course, mathematically, one could use either $\pm$ form in constructing a reciprocal adjustment;

$$\left(n \pm \frac{1}{n}\right) \cdot u \tag{107}$$

and let say, the shorter length be the reference, $L' = n \cdot u$, and the longer its apposite, $L = (n + 1/n) \cdot u$. We'd end up with essentially the same counts, n, but, of course, with slightly different units, u. But, then our reference lengths would be on the West and South, where we start the assignments, and this would be contrary to the art documented in the scriptures. Moreover, the divine ratio's hint, $\phi - 1/\phi = 1$, would not be as useful a 'clue' to us in driving our intuition towards the goal of recognition and recovery of the lost method. So, part of the puzzle in the ordering is understanding why the cardinal directions are used in the setting out of the variances in the design.

This puzzle inspires a number of ideas. Perhaps, placing the 28 division on the South/North parallels, and then changing it to the East/West parallels, was intended to suggest the idea of "rotation?" Or, perhaps, swapping the long and short sides from North/South to South/North was intended to hint at a possible "pole flip?" After all, if it's convention to have the North be the longer side, when applying the *principle of reciprocal adjustment*, why else would they have changed this to the South side in this instance? Why else would a conventional North, become represented by the South? It's hard to think of the Earth's rotational poles flipping North to South, but the Earth's magnetic poles have flipped several times as can be seen in the geological record of the magnetic orientation of the rocks at different strata. At any rate, the more we examine the Great Pyramid, the more we realise that the measurements and orientations, that we first think are errors in construction and mistakes by the builders, are actually intentional variances put there to encode some specific detail, and carry an important message. And when we account for this, we discover that it is modern man that has yet to measure the pyramid with a sufficiently high precision to discover the limits of the builders and where it is that they made actual errors. While Petrie seems to overlook these variances, taking them as representing the limits of the ancient builder's rough construction skills and obvious mistakes, Professor Piazzi Smyth, who surveyed Giza before him, took note of these issues, and was seemingly possessed of a more enlightened view, as J. Ralston Skinner writes,

> "(d.) Professor Smyth found a very curious feature as to the lengths of passage-ways, the proportions of the King's chamber, and of the coffer therein. He also infers the same as to the grand outlines of the pyramid base. It is:
> (1.) That a passage-way may present two lengths, the east side, say, being longer, or shorter, than the west side.
> (2.) That, compared with a perfect cubical chamber, the king's (and queen's) chamber is out of shape, or askew.
> (3.) That same happens as to the coffer. Very many circumstances connected with these irregularities of admeasurements would seem to point them as ***purposed*** , so that the differences between the measures of a standard cubical chamber, or coffer, and these as they are, served to give some derived result as the working out of a problem ; or else that a purposed variation in height or breadth might be intended to satisfy, in fact, two sets of measures differing from each other by some very slight quantity." – p.77, Construction of the Great Pyramid, Chapter III, Section I, "Key to the Hebrew-Egyptian Mystery in the Source of Measures (1894)," J. Ralston Skinner, ISBN 1-56459-519-6, Rare Esoteric Books, Kessinger Publishing, LLC.

> "(1.) Professor Smyth discovered a persistent variation in the lengths of passage-ways and dimensions of the chambers and of the coffer; that is, they all presented these variations on a mean of measure taken as a standard. He was forced to a conviction that this was purposed, and, if so, it was a scheme for the exhibition of variation of measures. The ability of the workmen to work to exactitude in these respects was amply displayed, and especially in the king's chamber, whose walls are in the hardest granite, polished to the highest reach of art, and whose joints are to be likened to the thickness of gold-leaf." – pp.111-112, Construction of the Great Pyramid, Chapter III, Section IV, "Key to the Hebrew-Egyptian Mystery in the Source of Measures (1894)," J. Ralston Skinner, ISBN 1-56459-519-6, Rare Esoteric Books, Kessinger Publishing, LLC.

and from these quotes, we see that the idea of ***deliberate*** variations in form and measure was at least present in the mind of one surveyor at Giza, even while others preferred to take these variances as evidence of limitations on the skill of the ancients.

We have an idea, that makes absolute sense, that *rough tools* limit one to *rough constructions* , and to obtain a high level of precision, requires several iterations in the refinements of the tools, over a rather long period of time, before one can tighten up the tolerances in the measurements of the layouts in actual construction. Our modern day tools have a decently high level of precision, but only because they have been revised many times over, each time increasing the precision of the tools, and using those higher precision tools themselves to create the next generation of even higher precision tools. It's not possible to 'jump' from a primitive stone ax, to immediately thereafter produce, for example, a high precision Japanese steel chef's knife. The amount of technological transformation between these events is enormous. So, it's natural for the modern surveyor to feel comfortable with the measurements that display apparent imperfections, consistent with his view that only rough and primitive tools were used to build the Pyramids, and to feel very uncomfortable to see and have to acknowledge that the precision required to construct one or more of these monuments is higher than even what we have available in the 21st century today. The latter idea doesn't fit with the linear progression model of historical development for mankind; Although, it fits the cyclical, advance..decline..advance, claims documented in the scripts of various religions. Knowledge and technologies are repeatedly being lost and re-discovered, in smaller and larger cycles of time, over long ages, so that no man can discover the true initial origins of any idea. (KJV, Ecclesiastes 1:9-11).

The uncovered length standard, $s = 32.59''$, then, implicated by the socket square side variances, represents some particular length standard other than the cubit. While it is on the order of magnitude of the cubit measures, like the royal cubit, $1cu = 20.632''$, it is nevertheless even longer than the long "sacred cubit" proposed by Sir Issac Newton, which is about, $1cu \approx 25''$, by a significant amount; and it is at the same time smaller than the often found standard "double cubit" rods which have length on the order of, $2cu = 41.2''$. Now, we notice that this socket square standard, s, is unequivocally some particular intermediate length between the "cubit" and the "double cubit."

Recall the *thesis* of *The Reciprocal Theory*, that it is the desired count, n, which is made precise by design, and the unit, u, that is left variable or inexact, to be only suggestive of what is being counted. So, the count, $n = 280$, is now determined, and the unit which is *about*, $s = 32.59''$, is now known, and this pair, (n, u), becomes a typical Giza riddle. What is it?

Well, the immediate obvious answer is that it is a *footstep* of a man.

Figure 7: *Hieroglyph for Walking*

The Egyptian hieroglyph for the concept of 'walking' consists of a pair of feet joined at the "*knees*" in a walking pose. Since a man's lower foot has length of about the same as his forearm, the glyph can be seen as the geometric figure of an isosceles triangle with the two equal sides being 1 cubit each in length. The maximum base length of such a triangle is, of course, then 2 cubits, when the angle is 180°, between these lower foot sides, and they become horizontal. The glyph then cleverly encodes the upper limit of a man's actual foot step length.

Of course, in reality, man uses his whole leg to walk, with the two length segments from hip to heal both participating in the action. But, by triangulating from the knee down to the heal only, the glyph is able to convey a particular idea about man's design.

Using Petrie's best value for the cubit, found in the King's Chamber dimensions, this glyph then tells us that, by design, the range of man's Step Length (SL) is about,

$$0 < \text{SL} < 41.264".$$

The walking hieroglyph is also used in Egyptian Mathematics as the symbol for *addition*. When turned around facing the other way, it represents *subtraction*. In one instance found in the *Moscow Papyrus Pr.14*, the same walking symbol is also used to indicate taking the *square* of a number†. So, *adding*, *subtracting*, and *squaring*, are all arithmetic operations denoted by the one and same walk symbol, as determined by the context. Hence, by the graphic depiction and the adapted usage, the ancient Egyptians signal the importance of the concept of man's foot step in their measurements and calculations, and we should therefore expect to find some evidence of this in the buildings and monuments they constructed.

† See p.178 in, "*Four Geometrical Problems from the Moscow Mathematical Papyrus*," Battiscombe Gunn, T. Eric Peet, 1929 Research Article, https://doi.org/10.1177/030751332901500130.

$\therefore$

The Foot Step

Now we easily recognise this value, $s = 32.59''$, as a 'typical' length of a man's footstep, that is, the length of the distance he pushes his foot forward one step at a time. The thing about man's footstep, however, is that it is a variable quantity. There is no one unique Step Length, $s = \mathrm{SL}$, and man's footstep varies with his height, speed of the walk, age of the individual, gender, health condition, nature of the terrain over which the walk is being done, whether the walk is being done with conscious voluntary control over its biomechanical dynamic parameters, or the walk is being undertaken without attention to conscious control, the so called "free walking" condition, and so on.

With the cubit, which is a measure of man's forearm, there is also variation among individuals. So, when a new Pharaoh ascends to the seat of power, since he sets and establishes the order and traditions, the calendar begins anew to record time from the first day of his rule, like in the 3rd month of the 2nd year of the reign of this or that Pharaoh etc. . . , and it is his arm that becomes the standard by which measures are established. He rules time and space.

In their woodcraft book[6], "*By Hand and Eye*," authors George R. Walker and Jim Tolpin explain that craftsmen didn't generally use fixed standard units of length when crafting furniture long ago. Artisans used the unmarked straight edge, rather than rulers with standard gradations, and employed dividers to construct *proportions*, instead of *measuring* out lengths. All measures were relative to man's body parts, via proportions, and depending on who you were making a chair or table for, that person's measures, or an approximation of them, would be used to design and build. That generally makes the furniture most comfortable for the user. Today, we would call that *customization*, a special consideration given to those users who can afford it, since we have become so accustomed to the more efficient and economical "*standards*" for everything we create.

In ancient Egypt, the whole environment of the Pharaoh would be made comfortable for him, so wherever he went in his land, he would find it suitable for his form. So, looking back at archaeological data, we find many cubit rods with different lengths. They are all about the length of the forearm, or related to it, but there is no exact consistency in the lengths. However, at least we can say that this particular cubit rod must be linked to that Pharaoh or his generation line, if his descendants decide to honor him by continuing to use the old standard. So, cubit standards can and do change, but at least the individual's arm length is fixed for his adult life, so that there's one standard obtainable from his forearm.

[6]Geo R. Walker and Jim Tolpin, "*By Hand & Eye*," Published by Lost Art Press, 2013, 5th Printing, ISBN: 0985077752. https://lostartpress.com/products/by-hand-eye-1 , https://www.byhandandeye.com/books/

Now, in the case of the Step Length, the same individual can exhibit many different step lengths. Indeed, the distance of the step is under voluntary control. However, there's a minimum value, which is near zero, $\mathrm{SL} = 0$, where he just inches forward a bit at a time, and for each individual there's his or her own personal maximum value, $\mathrm{SL} = \mathrm{SL}_{max}$, determined by how far apart he or she can easily and comfortably separate their two feet. That maximum value is the base of the isosceles triangle having the two feet for the equal sides, spread open at the widest practical angle possible for that person, that still enables them to execute the action of walking.

The architect here has selected a particular 'Step Length' of certain size, $\mathrm{SL} = 32.59''$, to match with his step count, $n = 280$, for the 'socket square'.

So, the pryamid side length is 280 steps, as measured by *one particular* foot step length, and the pyramid height is 280 cubits, as measured by *one particular* forearm length from elbow to fingertip. Both of these units, though different in length, are to be found encoded in the Great Pyramid construction itself. The cubit is roughly indicated in the Queen's Chamber side wall 5-level cutout niche, and its more exact value given with '*precision*' (as the covert '*two fingers*' reference encoded in the King's Chamber dimensions indicate; and the overt meticulously '*polished*' granite walls again visually immediately suggest), as Petrie remarks, in the more carefully constructed King's Chamber dimensions. While, the Step Length is encoded in the Great Pyramid's foundation sockets, and confirmed again in the subterranean chamber, as we shall see in the discussion below.

Of course, the pyramid isn't actually built to that height, $280\,cu$, and the pyramid base doesn't actually extend to that socket square, $280s \times 280s$, these are some of the "*reference metrics*," used to outline the extensions of the design plan. The pyramid height is truncated, by leaving off the capstone, and the "*casing square*" is about 1 footstep inside the socket square (this design is required to enable a man to walk around the pyramid base "on the socket square line," to verify that he can circumambulate the base 7 times in one hour, or travel 28 lengths of the side in just 1 hour of time. He can't walk exactly at the line where the casing meets the pavement, so he takes one step away from the true base to give himself space to actually walk, etc..—issues that will be dealt with elsewhere).

We don't have to introduce any externally determined units of measure from elsewhere, outside the Giza plateau, to evaluate the Pyramid construction. The builders already put the relevant units into the dimensions of the work for reference. The only external ideas required, are that the $20.632''$ length is *one particular* measure of a "man's forearm," and that the $32.59''$ length is *one particular* measure of a "man's footstep." These are things man is expected to notice, about himself, in order to "interpret" the meaning of the structure.

This appears to better explain Herodotus' cryptic statement that the height is the *same* as the length. Well, they have the same count of 280 units, but the units are different. The height is measured in cubits, as a man logically uses his hands to climb vertically up the cliff face of a mountain, and the base length is measured in foot steps, as a man logically pushes his foot horizontally as he walks about on the flat plane. A man pulls himself up 1 forearm distance at a time, naturally, using his arms, taking 280 repetitions of this discrete action to reach the summit, and a man steps forward 1 footstep at a time, naturally, using his feet on the level ground, taking 280 repetitions of that discrete action to travel the length of the side of the square base. The same number of 280 discrete actions measures both the length and the height, once the reference to the design of a man is understood, as his body is being used to measure.

One can imagine that when Herodotus querried the Egyptian Priests about the construction of the Great Pyramid, the Priests told him that the length is measured by foot, and the count in length and height are the same.

As A. W. Verrall says of Herodotus[7], "*His whole history of both pyramids, including the description and measurements, is given avowedly from information received, and even written mostly in the form of quotation*" (pg.198), and concerning the idea of the height being the same as the base side, "*There is nothing to show that it was his idea. He gives it simply as the statement of his informants...*" (pg.199).

But, whatever Herodotus thought he heard them say, and recorded from his memory, this idea, in the original ancient Greek, this statement has been subsequently translated into modern English variously as, "*Its base was square, each side eight hundred feet long, and its height is the same;*" (Godley[8], 1920, c.124); "*it is square, eight hundred feet each way, and the height the same.*" (Rawlinson, 1897, 208); "*Twenty years were spent in erecting the pyramid itself: of this, which is square, each face is eight plethra, and the height is the same*" (Cary[9], 1859, c.124); and, "*The pyramid itself is a work of twenty years; it is of a square form ; every front is eight plethra long, and as many in height;*" (Beloe[10], CXXIV, p.111).

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[7] A. W. Verrall (1898). *Herodotus on the Dimensions of the Pyramids.* The Classical Review,
12, pp 195-199 doi:10.1017/S0009840X00030559

[8] Book II c.124; "*Herodotus*," with an English translation by A. D. Godley. Cambridge. Harvard University Press. 1920.

[9] "*Herodotus. A New And Literal Version from the Text of Baehr. With A Geographical and General Index*," by Henry Cary, M.A., Worcester College, Oxford. 1859; Book II, c.124, p.145

[10] c.CXXIV, pp.111-112 in "*Herodotus, Translated from the GREEK with Notes and Life of the Author*," by the Rev. William Beloe, A New Edition, Corrected and Revised, Philadelphia: Published by McCarty & Davis, 171 Market Street; 1844.

One wonders, for example, where he got the idea of 800 feet? And did the idea of 800 feet, and the idea of length and height being in some way the same, come from the same conversation with the Priests? Herodotus does not say. But, in other places he claims he himself measured this or that structure and determined the number of feet, which we presume to be Greek Feet of his time. But, even the standard of the Greek Foot changed from one place to another, even in his own day. The likely scenario here, is that Herodotus himself estimated the length of the base to be 800 Greek Feet, and when writing up his notes, simply combined that with the information given him by the Priests that the height was the same, since he couldn't measure the height himself. There was no way to climb the smooth limestone outer casing that was there in his time. *(Apparently, the second pyramid, that of Khafre, did have foot holds cut out on the smooth stone casing surface leading all the way to the top of that pyramid, allowing individuals to climb to the top even when the casing stones were present. However, the Great Pyramid, attributed to Khufu, seemed to lack any such foot holds on its smooth limestone surface, at least no mention of such exists in the historical literature.)*

When the Priests told him the length was measured by the foot, he "heard" and interpreted that to mean the typical '*stationary foot*' measure from heal to toe, instead of the '*walking foot*' measure of the foot step length. Thinking this way, Herodotus then probably just went over to the Great Pyramid and measured to find out how many feet that length was. So, the germ of truth is contained in his cryptic text, but the ideas got mangled, probably because of the language differences and having to translate from Egyptian to Greek in communicating with the Egyptian Priests[11].

Perhaps too, the Priests were being coy with him, telling him a truth in an aphoristic way that they knew he would not understand. Everybody guarded their secrets in those days, and people did not give up their precious treasures easily. One had to earn the right to knowledge, by thinking for oneself. The Priests would be naturally reluctant to give this busybody foreigner, who came asking all these penetrating questions about their precious knowledge, the actual detailed information he sought. Yet, the Egyptian Priests would be constrained to tell the truth, given their religious principles, and belief in being judged in the underworld after this life. One can speak the truth, and yet convey a false impression, by relying on the lack of intelligence on the part of the listener to be able to discern the meaning. Jesus does this many times in the new testament. When his disciples asked him why he spoke in parables instead of speaking plainly, he said, "*Because it is given unto you to know the mysteries...but to them it is not given*"(KJV, Matthew 13:11; 13:34), and, "*Unto you it is given to know...but unto them that are without, all these things are done in parables:*

[11] Elsewhere Rawlinson comments on Herodotus' report of Cheops sending his daughter into prostitution to raise money for building the pyramid: "*we may be certain that Herodotus never received it from "the priests" whose language he did not understand, but from some of the Greek "interpreters," by whom he was so often misled.*" Note 1, Sec.126, pg.210, Book II.

That seeing they may see, and not perceive; and hearing they may hear, and not understand;" (Mark 4:11-12). Hence, this was evidently a practice of the Priesthood 2000 years ago, on how they would respond to 'outsiders' when communicating; a practice probably inherited from much further back in time to the days of Herodotus. So, what came out of the Priest's mouth was the truth. But, what entered into Herodotus' ear was just confusion; and it is this confusion that has usually left modern non-mathematician readers perplexed.

> *Subterranean Chamber*
>
> *Large chamber, E.Wall 325.9; at 100 from W.wall 329.6?; N.wall 553.5; S.wall 554.1* (Petrie: p.19 in 1990 ed; p.59 in 1883 ed)

Petrie says the width of Subterranean Chamber along the East Wall measures $325.9''$. The West Wall is obstructed as he writes, "*To measure along the W. side is impossible, without clearing away a large quantity of stones; and as there is no place to stack them safely without their going down the shaft, I could only measure the width at 100 from the W.end, perhaps somewhat askew.*" (p.20 in 1990 ed; p.60 in 1883 ed). So he measures 100 inches away from the West Wall to determine the width there, to get another alternate measure, and obtains $329.6''$, a measure which he says was probably "*askew*" (Yet, other authors have quoted the Width of this Subterranean Chamber as $330''$). Then he measures the length of the chamber, finding it is $553.5''$ on the North Wall, and $554.1''$ on the South Wall.

The first thing we notice, immediately, is that the '*East Wall*' of the Chamber is exactly 1 unit, s', of the measure found up on the ground level for the '*East Side*' of the Great Pyramid, that is encoded by the foundation sockets; East Wall corresponding to East Side, giving us confirmation and confidence in our model of reciprocal adjustment. That, $s' = 10s = 10 \cdot 32.59'' = 325.9''$, tells us that the Width of the chamber is 10 footsteps wide. Armed with this revelation, we calculate the length of the Chamber in footsteps, $553.5''/32.59'' = 16.98$, $554.1''/32.59'' = 17.00$, and find it's practically exactly 17 of the same footsteps.

So, the chamber is $10s \times 17s$. Notice that when we add the 2 units of measure found in the socket square above, $s' = 10s = 325.9''$, and, $s'' = 7s = 228.1''$, we get, $s' + s'' = 325.9'' + 228.1'' = 554.0''$, the very length of the chamber, which tells us that both these values found up on the ground level, s' and s'', are used to define the dimensions of the subterranean chamber. That room is, $(s') \times (s' + s'')$, which is, of course, $(10s) \times (10s + 7s)$, as we just found. All this provides additional confirmation that the North/South parallels in the socket square at ground level are indeed divided into that count, $n = 40$, which now gives us the unit, $s'' = 7s = 228.1''$, that contributes to the Subterranean Chamber design.

Thinking of The Great Pyramid as an intentional mathematical puzzle, accommodating the anticipated errors of re-measurement by future surveyors, there are *checks* incorporated into the structure to validate hypothetical notions formed from probable uncertain measurements. Here, in the subterranean chamber, we find a crosscheck of the counts, 28 and 40, found up on ground level in interpreting the 'socket square' measurements, so that the idea that these are the intended numerical values can be validated.

Socket Square Tolerances: Since the socket square employs the very same counts, 28 and 40, as the King's Chamber, we can simply use the previously derived formulas for the theoretical specification range, to see what is required in the measurements here. Now we no longer have to ground and polish walls to get the tolerance to fit the construction; the lengths here are on the order of 20 to 40 times that in the King's Chamber.

$$\frac{L}{1639.500} < \text{meas}\,\langle \Delta L(40)\rangle < \frac{L}{1559.500} \tag{49}$$

$$\frac{W}{811.500} < \text{meas}\,\langle \Delta W(28)\rangle < \frac{W}{755.500} \tag{50}$$

Here, the length parameters, L, L', once again represent the North/South data, $L = 9129.8'', L' = 9123.9''$, and the width parameters, W, W', again represent the East/West data, $W = 9130.8'', W' = 9119.2''$

$$\frac{9129.8''}{1639.500} < \text{meas}\,\langle \Delta L(40)\rangle < \frac{9129.8''}{1559.500} \tag{108}$$

$$5.568649'' < \text{meas}\,\langle \Delta L(40)\rangle < 5.854312'' \tag{109}$$

$$L - L' = 9129.8'' - 9123.9'' = 5.9'' \tag{110}$$

$$5.568649'' < 5.9'' \not< 5.854312 \quad \text{x} \tag{111}$$

$$\frac{9130.8''}{811.500} < \text{meas}\,\langle \Delta W(28)\rangle < \frac{9130.8''}{755.500} \tag{112}$$

$$11.251756'' < \text{meas}\,\langle \Delta W(28)\rangle < 12.085771'' \tag{113}$$

$$W - W' = 9130.8'' - 9119.2'' = 11.6'' \tag{114}$$

$$11.251756'' < 11.6'' < 12.085771'' \quad \checkmark \tag{115}$$

As we'd expect, the East/West data is a perfect fit, W, W', right in the middle of its expected theoretical specification range, so it gets our red check ✓ of approval. Of course, we should have anticipated that the North/South data would be out of range, L, L', since we saw, on discovery of the count, that it indicates, $n = 39$, hence the red x, instead. We suggested this should really be, $n = 40$, and have shown that this choice is consistent with the measurements of the length and width of the *subterranean chamber*; confirming an error in Petrie's data. But, if we fixed the South side data, $L' = 9123.9'' + .2'' = 9124.1''$, to account for Petrie's $5mm$ error, i.e. $< |\pm .3''|$, we'd then be within spec;

$$L - L' = 9129.8'' - 9124.1'' = 5.7'' \tag{116}$$

$$5.568649'' < 5.7'' < 5.854312 \ \checkmark \tag{117}$$

A more complete Precision Analysis of the socket square is in APPENDIX A.

The Azimuths. It has been noted, by several observers, that despite the unequal sides of the nominally apparent rectilinear *socket square*, the diagonals nevertheless seem to intersect at right angles, when one would expect these lines to cross at something much different, since the socket square isn't actually square, but rather more properly considered an irregular four sided polygon, with all four sides being different one from another. This prompts us to look again at Petrie's data, this time with a bit more care, to see if we can understand what is going on at that Pyramid base. So, here's a table of Petrie's base squares, once again, this time *including* the azimuth data.

	Core Square:		Casing Square:		Socket Square:	
	SIDES.	AZIMUTHS.	SIDES.	AZIMUTHS.	SIDES.	AZIMUTHS.
E.	$8999.4''$	$-5'26''$	$9067.7''$	$-3'57''$	$9130.8''$	$-5'21''$
W.	$9002.5''$	$-5'39''$	$9068.6''$	$-3'54''$	$9119.2''$	$-7'33''$
N.	$9002.3''$	$-4'35''$	$9069.4''$	$-3'20''$	$9129.8''$	$-3'20''$
S.	$9001.7''$	$-5'23''$	$9069.5''$	$-3'41''$	$9123.9''$	$+1'15''$
Mean	$9001.5''$	$-5'16''$	$9068.8''$	$-3'43''$	$9125.9''$	$-3'45''$
Mean Diff.	$1.0''$	$20''$	$.65''$	$12''$	$4.4''$	$2'42''$

(pp.38-39, Petrie 1883 ed; p.11, 1990 ed.)

Petrie notes that the whole *core square* of the pyramid seems to be off the true cardinal directions by almost a consistent amount, averaging about $5'16''$ of arc, with an anti-clockwise rotation; which he represents with '$-$' signs. So, instead of aligning true North, the Pyramid core is rotated Westward, facing off a little North-West from the current North Pole. But, apart from that, the sides are more square than the *socket square*. It is as if the earth shifted, after the builders had erected the core, resulting in the direction of the true North being displaced by that $5'^{+}$ arc movement. When we examine the *casing square*, once again we find it is more square'd than the *socket square*, but this time the entire casing square is rotated anti-clockwise by average of only $3'43''$. The immediate

thought that occurs, to the casual observer, is that the *core* must have been built at a time when the North Pole was aligned perfectly with the *core square's* orientation. Then a major cataclysm occurred, and shifted the land mass on which the pyramid sits, "*and then*" the casing was put on the pyramid, once again aligning to the then present true North. Thereafter "*another*" cataclysm occurred, shifting the pyramid once again, in the same anti-clockwise fashion. So that today, what we're looking at, is the result of at least two major land mass movements that must have occurred since the foundations were put in place. The recent casing that was present, then, must have been a repair job, after the first cataclysm destroyed the outer layer of the pyramid; and the last time the Egyptians worked on this pyramid, several millennia ago, they were just restoring it for posterity.

We are now in the process of restoring the pyramid once again. Currently, only the insides are being *shored up*, prompted by the profit of tourism; but a few casing stones on the outside at the base appear to have been repaired and restored to give the tourists a feel for what they looked like when first constructed. But, once it is understood what the pyramid represents, there's a good chance that men will once again fully restore the outer casing, and align it with the prevailing true North whenever this project is undertaken. The Great Pyramid is then seen as a clock, and recording device, marking and keeping time between major cataclysms, perpetually being damaged by nature, and restored by man, walking slowly to the left of North, as the Earth's outer crust shifts tick by tick over the Earth's inner core, during the long time frame of the ages.

The three large Giza pyramids even seem to sit on a directionally oriented spiral curve, suggesting the direction of the Earth's outer crust movement, consistent with the same 'westward' displacement of the Great Pyramid's own base squares, as if confirming the direction of the angular anti-clockwise advancement. The Giza Pyramids would then truly be immortal constructions, justifying the Arab saying, "*Man fears time, but time fears the Pyramids!*" For as long as man's existence is present on Earth, the pyramids would continue to be periodically restored. This is the thought the data inspires, at least, to the casual observer.

Now the experts, who are the professional Egyptologists and Archaeologists, tell us that this pyramid is only about 4500 years old, and there are no such cataclysms in that short time period to account for these displacements that the data exhibits. Besides this, the *socket square* azimuths are all over the place; at least, it seems that way at first glance. So, we must look again, at the data, to see if we can find an alternative explanation for these apparent aberrations.

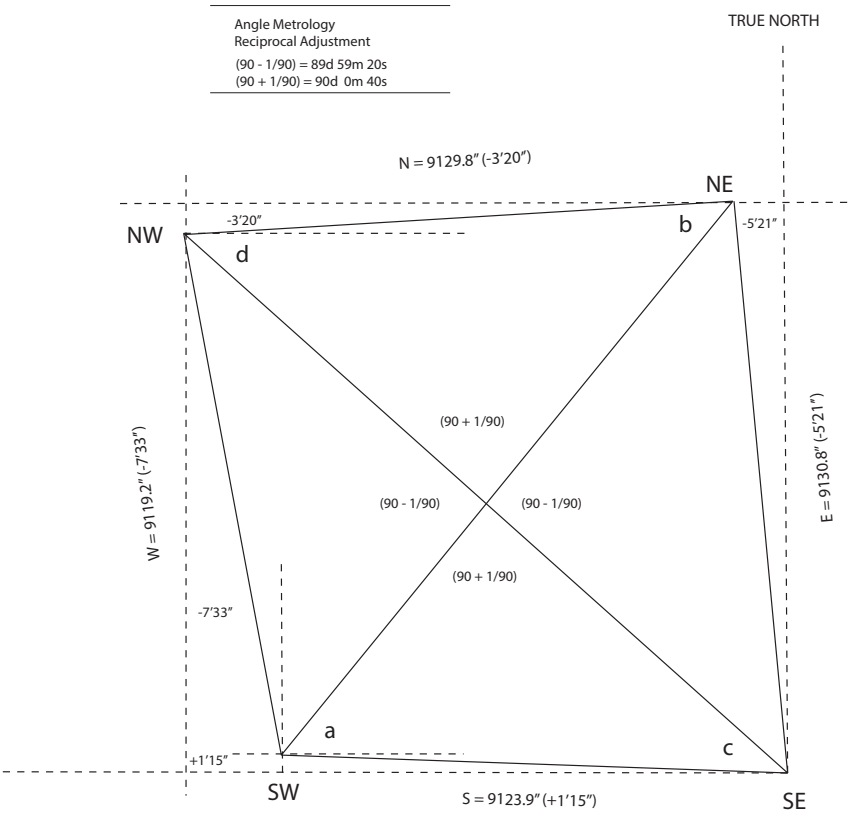

Figure 8: *The Great Pyramid Socket Square Diagonals (not drawn to scale). This is a greatly exaggerated distortion of the actual socket square to illustrate its deviations. The labels: a, b, c, d, are angles the diagonals make with the true East-to-West line parallels.*

The Crossing Diagonals. The angles at the crossing diagonals are very nearly but not exactly 90°, instead one pair of right angles appears to be modified by the usual *principle of reciprocal adjustment*, so that the East base side is actually subtended by a slightly smaller angle of, $(90 - 1/90)^\circ = 89^\circ 59' 20''$. This, however, suggests to us that the system of angle measures used is consistent with that which divides a right angle into 90 equal parts, or a circle into 360 equal parts. We know of this today as the system of degrees, which has been inherited from ancient times without any documented historical explanation of why this was considered useful. With two angles constructed as, $(90 - 1/90)^\circ$, and the other two angles constructed as the complementary, $(90 + 1/90)^\circ$, all four angles sum to the usual 4 right angles giving, $89^\circ 59' 20'' + 90^\circ 0' 40'' + 89^\circ 59' 20'' + 90^\circ 0' 40'' = 360^\circ 0' 0''$. So, in this case, the reference angle, $90 \cdot u$, is '*implied*,' and the apposite, $(90 - 1/90) \cdot u$, is what is actually measured. That unit, $u = 1^\circ$, by convention.

$$
\begin{aligned}
a = &\quad \tan^{-1}\left(\tfrac{\mathbf{E}\cdot\cos(5'21'')-\mathbf{S}\cdot\sin(1'15'')}{\mathbf{S}\cdot\cos(1'15'')-\mathbf{E}\cdot\sin(5'21'')}\right) &&= 45°03'21.11'' \\
b = &\quad \tan^{-1}\left(\tfrac{\mathbf{W}\cdot\cos(7'33'')+\mathbf{N}\cdot\sin(3'20'')}{\mathbf{N}\cdot\cos(3'20'')-\mathbf{W}\cdot\sin(7'33'')}\right) &&= 45°03'26.54'' \\
c = &\quad \tan^{-1}\left(\tfrac{\mathbf{W}\cdot\cos(7'33'')+\mathbf{S}\cdot\sin(1'15'')}{\mathbf{S}\cdot\cos(1'15'')+\mathbf{W}\cdot\sin(7'33'')}\right) &&= 44°55'58.00'' \\
d = &\quad \tan^{-1}\left(\left(\tfrac{\mathbf{E}\cdot\cos(5'21'')-\mathbf{N}\cdot\sin(3'20'')}{\mathbf{N}\cdot\cos(3'20'')+\mathbf{E}\cdot\sin(5'21'')}\right)\right) &&= 44°55'50.79''
\end{aligned}
$$

$$
\begin{array}{lllll}
a+c = & 45°03'21.11'' + 44°55'58.00'' & = 89°59'19.11'' & \approx (90-1/90)° \\
b+d = & 45°03'26.54'' + 44°55'50.79'' & = 89°59'17.33'' & \approx (90-1/90)° \\
a+d = & 45°03'21.11'' + 44°55'50.79'' & = 89°59'11.90'' & \approx (90-1/90)° \\
b+c = & 45°03'26.54'' + 44°55'58.00'' & = 89°59'24.54'' & \approx (90-1/90)° \\
 & & & \\
avg. = & & = 89°59'18.22'' & \approx 89°59'20'' \\
err. = & 1.78'' < 2'' & &
\end{array}
$$

In a perfect geometric construction, the angle at the diagonal crossing which subtends the East side, being the external angle of the adjacent triangle in the South, has to have, by the external angle triangle law, the same measure as the sum of the opposite inside angles, $a+c$, and again being adjacent to the triangle in the North, must also have the same measure as the sum, $b+d$, and both these sums appear to effectively display the value of, $(90-1/90)°$. The diagonal crossing angle in the quadrant with the West side, being exactly opposite at the crossing point, is also measured by these two sums. Then, the other two angles at the crossing, subtending the North Side, and subtending the South Side, are again equal to each other, and now measured by, $(90°-a)+(90°-d) = 180°-(a+d)$, and, $(90°-b)+(90°-c) = 180°-(b+c)$, both of these turning out to have values near, $(90+1/90)° = 180°-(90-1/90)°$. So, the 4 angle pair sums, $a+c$, $b+d$, $a+b$, and, $b+c$, are all expected to have the same value, $(90-1/90)° = 89°59'20''$. This gives us 4 values computed from Petrie's raw data, to attempt to verify the "thesis" that the *principle of reciprocal adjustment* was also used on the diagonal crossing angles to construct this "*tweaked*" socket square, possibly suggesting the angle metrology in use by the builders to be that which divides the right angle into 90 equal parts, and hence the circle into 360.

With an imperfect construction, to include variances and errors of measure, we separate the four triangles in these four quadrants, to treat them independently, and so we define the EAST ANGLE at the crossing to be that apex angle of the triangle whose base is the East Side with base angles, $(90-b)$ and $(90-c)$, so that the apex angle at the crossing is, $180-((90-b)+(90-c)) = b+c$; and the WEST ANGLE at crossing subtends the West Side with base angles, $(90-a)$ and $(90-d)$, with the apex angle at crossing being independently computed as, $180-((90-a)+(90-d) = a+d$; then, the NORTH ANGLE at crossing subtends the North Side with triangle base angles, d and b, making the apex

angle there, $180 - (b + d)$, which calculation we'll refer to by the complement, $b + d$; and the SOUTH ANGLE at crossing subtends the South Side with base angles, a and c, giving apex angle there, $180 - (a + c)$, which we'll again refer to by the complement angle, $a + c$.

Despite the fact that these results look close, and are very suggestive of the idea this diagonal crossing angle is intended to be that, $(90 - 1/90)°$, it can be proven geometrically that, given Petrie's particular raw survey data, the closest the quadrilateral's shape can ever come to manifest that angle, $\Theta = \angle(E)$, is about, $(90 - 1/86)°$. It is, in fact, locked into a narrow range of approximately, $90 - 1/2 < \Theta < 90 - 1/87$, and Petrie's data would have to be modified too much to get that fit, taking it outside the bounds of what we might consider reasonable. Intuitively, we can understand this by considering when the quadrilateral is a perfect square, and we shift the shape by a shear into a rhombus. No matter the shear transformation, the diagonals are always locked to cross at exactly 90°. For a quadrilateral to change this angle, the lengths of its sides must differ. But, the socket square is "almost" a square, so it stays close to that 90° crossing angle, only changing a bit, due to the unequal "tweaked" side lengths. Yet, the "tweaking" done here is not sufficient to include the exact metric, $(90 - 1/90)°$, in this particular quadrilateral. Although, it can be thought to *hint* at that metric. So, we must look elsewhere for alternative clues to the representation of angle data in the monument, if such exists.

Adjacent Apposites.

The question naturally arises, just as the *subterranean chamber's* dimensions can be seen as a *check* for the metrics uncovered above on the socket square, is there a similar *check* in the construction that would allow us to determine similarly the *angle metrics* somehow? Can we confirm that 90° metric that seems *hinted at* by the diagonal crossing results, and disambiguate between it and other metrics that might equally play the role there?

Now, while it seems obvious that *The Reciprocal Theory* would naturally employ the sides *opposite*, for the *apposites*, as we have done so far in our analysis above, there's nothing that says we can't use any two similar lengths, the one slightly different from the other, anywhere in our design as candidates to encode the local metrical parameters; as long as it's somewhat obvious to the subsequent surveyor that these choice lengths are to be related in the measurement; Especially in a modified square, which has all sides about the same length, there are other selections to the parallel motif available. An angle, after all, is the metric that determines the relative orientation of two *adjacent* lines that meet at a point, or vertex. That gets us thinking that we should look at sides *adjacent* for the *apposites*, when it comes to ways of encoding angle count values, or counts to act as *checks* for angles.

Taking a quick look at adjacent pairs on the square;

$$(E,N): \quad n = 1/\sqrt{1-9129.8''/9130.8''} = 95.56 \rightarrow 96 \tag{118}$$
$$(E,S): \quad n = 1/\sqrt{1-9123.9''/9130.8''} = 36.38 \rightarrow 36 \tag{119}$$
$$(S,W): \quad n = 1/\sqrt{1-9119.2''/9123.9''} = 44.05 \rightarrow 44 \tag{120}$$
$$(N,W): \quad n = 1/\sqrt{1-9119.2''/9129.8''} = 29.35 \rightarrow 29 \tag{121}$$

But, then we recall the North/South raw data has a problem. It suggests, $n = 39$, whereas we're certain the neighbor, $n = 40$, is the correct count that should appear there. We saw that we could either adjust the North data down by $0.2''$, from $9129.8''$ to a corrected value, $9129.6''$, or alternatively, we could adjust the South data up by $0.2''$, from $9123.9''$, to a modified, $9124.1''$. So, let's see what these changes would suggest.

$$(E,N): \quad n = 1/\sqrt{1-9129.6''/9130.8''} = 87.23 \rightarrow 87 \tag{122}$$
$$(E,S): \quad n = 1/\sqrt{1-9124.1''/9130.8''} = 36.92 \rightarrow 37 \tag{123}$$
$$(S,W): \quad n = 1/\sqrt{1-9119.2''/9124.1''} = 43.15 \rightarrow 43 \tag{124}$$
$$(N,W): \quad n = 1/\sqrt{1-9119.2''/9129.6''} = 29.63 \rightarrow 30 \tag{125}$$

Going back to the sacred order recorded in Ezekiel, we have, E-N-S-W-E, in cyclic fashion, so the neighbor pairs there are : (E,N), (N,S), (S,W), (E,W), but two are missing, since, (N,W) and (E,S), are not pairs of lengths following each other in this measuring ritual. The (E,W) and (N,S) are the references with opposite parallel apposite, which we've dealt with at length above. So, there are only two remaining which are the references with adjacent apposites, approved by the measuring ritual, and these are: (E,N) and (S,W). From the corrections required in the North/South raw data, we see that there's some flexibility here, as it appears the count for the (E,N) pair falls in the range,

$$87 \leq n(E,N) \leq 96 \tag{126}$$

and since this is a corner of a square, which under the Babylonian sexagesimal system, would normally be 90°, we wonder if a division count, $n = 90$, might work here. So, we test,

$$E \times (1 - 1/90^2) = N\ ? \tag{127}$$
$$9130.8'' \times (1 - 1/90^2) = 9129.67'' = 9129.8'' - 0.13'' \tag{128}$$

$$9129.6'' < 9129.67'' < 9129.8'' \tag{129}$$

So, we realise that it is certainly possible, that a count, $n = 90$, could be encoded for the adjacents at this corner of the square, since we can split that

$0.2''$ adjustment required, between the North data and South data to tidy up and correct the data. Then, we notice that the (S,W) pair is just off from 45, by 1 or 2, depending on the adjustment. So, we think, perhaps the four counts encoded in the socket square are intended to be, 28, 40, 45, 90; two from the opposite apposites and two from the adjacent apposites. When we sum these whole numbers up;

$$28 + 40 + 45 + 90 \;=\; 203 \quad ! \tag{130}$$

But, that 203 is the number steps in the Great Pyramid to the truncated top. Now this does begin to look like it could be that *check* we're looking for (We only need a single parameter, M, to suggest the angle metrics, and there's only one value that fits the equation, $28+40+M/2+M = 203$. That's, $M = 90$); A simple numerical clue to aid disambiguation among all the nearly equally valid candidates; making *exact* from what is otherwise *imprecise*.

Thus we would have,

$$\begin{array}{lllll}
E & = & & = 9130.8'' & = 9130.80'' - 0.00'' \\
N & = (1 - \frac{1}{90^2}) \cdot E & & = 9129.67'' & = 9129.80'' - 0.13'' \\
S & = (1 - \frac{1}{40^2}) \cdot N & = (1 - \frac{1}{40^2}) \cdot (1 - \frac{1}{90^2}) \cdot E & = 9123.97'' & = 9123.90'' + 0.07'' \\
W & = (1 - \frac{1}{45^2}) \cdot S & = (1 - \frac{1}{45^2}) \cdot (1 - \frac{1}{40^2}) \cdot (1 - \frac{1}{90^2}) \cdot E & = 9119.46'' & = 9119.20'' + 0.26'' \\
W & = (1 - \frac{1}{28^2}) \cdot E & & = 9119.15'' & = 9119.20'' - 0.05''
\end{array}$$

There are two paths here to obtain the West Side length: the indirect path, $E \to N \to S \to W$, which gives us, $W = 9119.46''$; and the more direct path, $E \to W$, yielding, $W = 9119.15''$, the difference is, $9119.46'' - 9119.15'' = 0.31''$. This is right in line with Petrie's own reported survey error, $\pm 0.3''$. Note that,

$$0.99875816278 \;=\; (1 - 1/45^2) \cdot (1 - 1/40^2) \cdot (1 - 1/90^2) \tag{131}$$

$$0.99872448980 \;=\; (1 - 1/28^2) \tag{132}$$

$$0.00003367298 \;=\; (1 - 1/45^2) \cdot (1 - 1/40^2) \cdot (1 - 1/90^2) - (1 - 1/28^2) \tag{133}$$

and we can never close the path perfectly, since, $9000'' \times 0.0000337 = 0.3''$, and this is fixed regardless of juggling the data number precision values to account for various types of rounding effects. We have two numbers, $(28, 40)$, which we can consider *lineal* counts, since they emerge unequivocally from nominally parallel lines, and we have two numbers, $(45, 90)$, which we can consider *angular* counts, since they are *suggested* by adjacent lines that intersect at a nominal right angle on the modified square. These pairs are in some sense duals of each other, since, $45 = 90/2$ and $90 = 2 \cdot 45$, and, $28 \approx 40/\sqrt{2}$, and $40 \approx 28 \cdot \sqrt{2}$, that is, the lineal pair arise naturally from an oblong rectangle that is $1/2$ as wide as it is long, and one angle reference is also $1/2$ a right angle, which is the other angle in the pair. The main angle count, then, is, $n = 90$, as it matches the usual convention for right angles, and notice that it emerges from the two

adjacent sides whose lengths are the smallest difference from one another. This makes sense, since the reciprocal adjustment is, L/n^2, and all the other $n's$ in the system here are smaller than, $n = 90$. Now, each side on the socket square can be associated with a pair of numbers, and the LCM on the lengths in the four cardinal directions can be assigned a composite count as before. This gives us the 360 count, for the circle, and the 1260 count, as in the biblical $3\frac{1}{2}$ years of 360 days, or 42 months.

$$\begin{array}{lll} E & = (28, 90) & \rightarrow \text{ LCM}(28, 90) = 1260 \\ N & = (90, 40) & \rightarrow \text{ LCM}(90, 40) = 360 \\ S & = (40, 45) & \rightarrow \text{ LCM}(40, 45) = 360 \\ W & = (45, 28) & \rightarrow \text{ LCM}(45, 28) = 1260 \end{array}$$

We have to make a decision on which West side calculation to use, and we pick the more direct formula, since the long path to get there turns about this way and that to capture information about the angle count, and we only really need the first angle count, $n = 90$, we meet at the East-to-North junction, to know what count divides the circle. The South-to-West vertex is nominally, $n = 45$, symbolically, to fit the "doubling" theme, but it neither looks like a 45° angle, nor does it fit the path the best, preferring a fit to one off, $n = 44$. The best we can say is that it fits within that $\pm 0.3''$ error from Petrie's survey. But we are attempting to 'improve' on Petrie's data, by using these clues from *The Reciprocal Theory* to effectively upgrade his $0.1''$ to $0.01''$ without having to re-measure the socket square at Giza.

Our revised and improved system is therefore:

$$E = 1 \cdot E \tag{134}$$

$$N = (1 - \frac{1}{90^2}) \cdot E \tag{135}$$

$$S = (1 - \frac{1}{40^2}) \cdot N \tag{136}$$

$$W = (1 - \frac{1}{28^2}) \cdot E \tag{137}$$

$$\text{so, roughly:} \quad s = \frac{9130.80''_{\text{E}} + 9119.15''_{\text{W}} + 9129.67''_{\text{N}} + 9123.97''_{\text{S}}}{4 \times 280} = 32.593'' \tag{138}$$

$$\text{or, more precisely:} \quad s_\circ = \frac{9130.80''_{\text{E}} + 9119.15''_{\text{W}} + 9129.67''_{\text{N}} + 9123.97''_{\text{S}}}{4 \times 280 - 10/28 - 7/40} = 32.608'' \tag{139}$$

All side lengths of the Socket Square are fully determined by the East side length taken as our primary reference, along with the three counts, 90, 40, 28,

and *The Reciprocal Theory*.

Data correction applied: $9130.8''\text{E} \to 9139.80''\text{E}$, i.e. synthetically upgraded to $.01''$ precision; $9119.2''\text{W} \to 9119.15''\text{W}$, i.e. improved precision to $.01''$; $9129.8''\text{N} \to 9129.67''\text{N}$, i.e. reduced by $0.13''$ and upped to $.01''$ precision; and, $9123.9''\text{S} \to 9123.97''\text{S}$, i.e. increased by $0.07''$ and upped to $.01''$ precision. The unit of the footstep, rough, $s = 32.593''$, and, precise, $s_\circ = 32.608''$, remain the same as before to the 3 decimal places, even with all these small changes, so previous conclusions concerning them remain unchanged.

Note that if the objective was just to 'close the path', so that there would be no gap on the West side from indirect and direct paths to get there, the optimal angle check integers would be, $(44, 86)$, instead of, $(45, 90)$. So, there are a few choices that would give better results in miminizing the gap there. But, only one satisfies the check expression, $28 + 40 + M/2 + M = 203$.

sum	check	$M/2$?	gap err
$28 + 40 + 44 + 92$	$= 204 \neq 203$	$, 44 \neq 92/2$	$+0.15''$
$28 + 40 + 44 + 91$	$= 203$?	$, 44 \neq 91/2$	$+0.13''$
$28 + 40 + 45 + 90$	$= 203$	$, 45 = 90/2$	$+0.31''$
$28 + 40 + 44 + 90$	$= 202 \neq 203$	$, 44 \neq 90/2$	$+0.10''$
$28 + 40 + 44 + 88$	$= 200 \neq 203$	$, 44 = 88/2$	$+0.05''$
$28 + 40 + 44 + 87$	$= 199 \neq 203$	$, 44 \neq 87/2$	$+0.02''$
$28 + 40 + 44 + 86$	$= 198 \neq 203$	$, 44 \neq 86/2$	$-0.00''$
$28 + 40 + 43 + 86$	$= 197 \neq 203$	$, 43 = 86/2$	$-0.23''$

Socket Square Walk Path

Now let's become more familiar with Petrie's survey data set, and its various implied derived metrics. First, we look at how well Petrie's raw data fits the closed quadrilateral, when the survey lengths and azimuths are used to compute the walk path around the tweaked socket square.

We set the S.W. corner of the socket square at the "scored line," which, as we remarked before, is *chiseled* into the actual South West foundation socket rock cutout, some $17.5''$ East of the socket's actual West end, as this is Petrie's definition of the corner there, as our start point; i.e. let the coordinate pair, $(East, North) = (0, 0)$, be our origin. Then, we compute and check,

$$
\begin{aligned}
Path &= (\ +S\cdot cos(1'15'')\ ,\ -S\cdot sin(1'15'')\quad) \\
&+ (\ -E\cdot sin(5'21'')\ ,\ +E\cdot cos(5'21'')\quad) \qquad (140)\\
&+ (\ -N\cdot cos(3'20'')\ ,\ -N\cdot sin(3'20'')\ ) \\
&+ (\ +W\cdot sin(7'33'')\ ,\ -W\cdot cos(7'33'')\ ) \\
&= (\ +errEast\ ,\ +errNorth\ ) \qquad (141)\\
&= (\ 0\ ,\ 0\ )\ ?
\end{aligned}
$$

Plugging in the $0.1''$ precision raw *length* data from the survey, and comparing with the $0.01''$ precision model corrected *length* data, on the same path formula with both using Petrie's *azimuth* data,

$$
\begin{aligned}
Path &= (+9123.9''\cdot cos(1'15''), -9123.9''\cdot sin(1'15'')) \\
&+ (-9130.8''\cdot sin(5'21''), +9130.8''\cdot cos(5'21'')) \qquad (142)\\
&+ (-9129.8''\cdot cos(3'20''), -9129.8''\cdot sin(3'20'')) \\
&+ (+9119.2''\cdot sin(7'33''), -9119.2''\cdot cos(7'33'')) \\
&= (-0.0786'', -0.5591'') \qquad (143)
\end{aligned}
$$

$$
err = \sqrt{\left((-0.0786'')^2 + (-0.5591'')^2\right)} = 0.565'' \approx \frac{1}{2}'' \qquad (144)
$$

: New path with corrected data: $\rightarrow$

$$
\begin{aligned}
Path &= (+9123.97''\cdot cos(1'15''), -9123.97''\cdot sin(1'15'')) \\
&+ (-9130.80''\cdot sin(5'21''), +9130.80''\cdot cos(5'21'')) \qquad (145)\\
&+ (-9129.67''\cdot cos(3'20''), -9129.67''\cdot sin(3'20'')) \\
&+ (+9119.15''\cdot sin(7'33''), -9119.15''\cdot cos(7'33'')) \\
&= (+.12138'', -0.50901'') \qquad (146)
\end{aligned}
$$

$$
err = \sqrt{\left((.12138'')^2 + (-0.50901'')^2\right)} = 0.523'' = .565'' - .04'' \qquad (147)
$$

We notice that Petrie's survey data doesn't close the path perfectly to bring us back to the origin, $(0,0)$; instead, he's a little off by about $1/2$ of an inch in a direction somewhat South West of the start point. We wonder if this is

the source of his estimate of that claimed $\pm.5''$ error that he gives for the error attributed to the settling of the rocks and movement due to earthquakes on the Giza plateau introducing some displacements that produce errors from the original construction. At any rate, our model correction does slightly better by about 4/100 of an inch closer, although the return point is now somewhat in the North West direction. Nevertheless, both Petrie's $0.1''$ and the Model's $0.01''$ data produce similar results on the path, as both are about 1/2 inch off.

Socket Square Diagonals

Next we become familiar with diagonal lengths implied by Petrie's survey data, and the model, so far in our analysis. Let the diagonal length from the SE corner to the NW corner be, D_1, and the diagonal length from the NE corner to the SW corner be, D_2. We compute the values of these lengths.

There are "two ways" to compute the diagonal length, D_1, as we can do so from the perspective of the South West corner, $\angle(S, W)$, or from the perspective of the North East corner, $\angle(N, E)$. Similarly, to compute the diagonal length, D_2, we can start from the South East corner, $\angle(S, E)$, or again from the North West corner, $\angle(N, W)$.

PETRIE'S $0.1''$ SURVEY DATA:

$$
\begin{aligned}
D_1 &= \sqrt{((9123.9 - 9119.2 \cdot \cos(\angle(S,W)))^2 + (9119.2 \cdot \sin(\angle(S,W)))^2)} \\
D_1 &= \sqrt{((9123.9 - 9119.2 \cdot \cos(90 + 1'15'' + 7'33''))^2 + (9119.2 \cdot \sin(90 + 1'15'' + 7'33''))^2)} \\
D_1 &= \sqrt{((9123.9 - 9119.2 \cdot \cos(324528/3600))^2 + (9119.2 \cdot \sin(324528/3600))^2)} \\
D_1 &= \sqrt{((9123.9 - 9119.2 \cdot \cos(90.146667))^2 + (9119.2 \cdot \sin(90.146667))^2)} \\
D_1 &= 12916.3201598 = 1.41458800541 \cdot 9130.8
\end{aligned}
$$

$$
\begin{aligned}
D_1 &= \sqrt{((9129.8 - 9130.8 \cdot \cos(\angle(N,E)))^2 + (9130.8 \cdot \sin(\angle(N,E)))^2)} \\
D_1 &= \sqrt{((9129.8 - 9130.8 \cdot \cos(90 - 3'20'' + 5'21''))^2 + (9130.8 \cdot \sin(90 - 3'20'' + 5'21''))^2)} \\
D_1 &= \sqrt{((9129.8 - 9130.8 \cdot \cos(324121/3600))^2 + (9130.8 \cdot \sin(324121/3600))^2)} \\
D_1 &= \sqrt{((9129.8 - 9130.8 \cdot \cos(90.033611))^2 + (9130.8 \cdot \sin(90.033611))^2)} \\
D_1 &= 12915.9808574 = 1.41455084521 \cdot 9130.8
\end{aligned}
$$

$$
\Delta(D_1) = 12916.3201598 - 12915.9808574 = 0.339302''
$$

$$
\begin{aligned}
D_2 &= \sqrt{((9123.9 - 9130.8 \cdot \cos(\angle(S,E)))^2 + (9130.8 \cdot \sin(\angle(S,E)))^2)} \\
D_2 &= \sqrt{((9123.9 - 9130.8 \cdot \cos(90 - 1'15'' - 5'21''))^2 + (9130.8 \cdot \sin(90 - 1'15'' - 5'21''))^2)} \\
D_2 &= \sqrt{((9123.9 - 9130.8 \cdot \cos(323604/3600))^2 + (9130.8 \cdot \sin(323604/3600))^2)} \\
D_2 &= \sqrt{((9123.9 - 9130.8 \cdot \cos(89.890000))^2 + (9130.8 \cdot \sin(89.890000))^2)} \\
D_2 &= 12895.6263263 = 1.41232162858 \cdot 9130.8
\end{aligned}
$$

$$
\begin{aligned}
D_2 &= \sqrt{((9129.8 - 9119.2 \cdot \cos(\angle(N,W)))^2 + (9119.2 \cdot \sin(\angle(N,W)))^2)} \\
D_2 &= \sqrt{((9129.8 - 9119.2 \cdot \cos(90 + 3'20'' - 7'33''))^2 + (9119.2 \cdot \sin(90 + 3'20'' - 7'33''))^2)} \\
D_2 &= \sqrt{((9129.8 - 9119.2 \cdot \cos(323747/3600))^2 + (9119.2 \cdot \sin(323747/3600))^2)} \\
D_2 &= \sqrt{((9129.8 - 9119.2 \cdot \cos(89.929722))^2 + (9119.2 \cdot \sin(89.929722))^2)} \\
D_2 &= 12896.0775244 = 1.41237104354 \cdot 9130.8
\end{aligned}
$$

$$
\Delta(D_2) = 12895.6263263 - 12896.0775244 = -0.451198''
$$

$$
: \quad 19.90'' < |D_1 - D_2| < 20.69''
$$

$$
D_1 > D_2
$$

PETRIE'S AZIMUTH DATA combined with the MODEL Corrected $0.01''$ Length DATA:

$$
\begin{aligned}
D_1 &= \sqrt{((9123.97 - 9119.15 \cdot \cos(\angle(S,W)))^2 + (9119.15 \cdot \sin(\angle(S,W)))^2)} \\
D_1 &= \sqrt{((9123.97 - 9119.15 \cdot \cos(90 + 1'15'' + 7'33''))^2 + (9119.15 \cdot \sin(90 + 1'15'' + 7'33''))^2)} \\
D_1 &= \sqrt{((9123.97 - 9119.15 \cdot \cos(324528/3600))^2 + (9119.15 \cdot \sin(324528/3600))^2)} \\
D_1 &= \sqrt{((9123.97 - 9119.15 \cdot \cos(90.146667))^2 + (9119.15 \cdot \sin(90.146667))^2)} \\
D_1 &= 12916.3343421 = 1.41458955864 \cdot 9130.80
\end{aligned}
$$

$$
\begin{aligned}
D_1 &= \sqrt{((9129.67 - 9130.80 \cdot \cos(\angle(N,E)))^2 + (9130.80 \cdot \sin(\angle(N,E)))^2)} \\
D_1 &= \sqrt{((9129.67 - 9130.80 \cdot \cos(90 - 3'20'' + 5'21''))^2 + (9130.80 \cdot \sin(90 - 3'20'' + 5'21''))^2)} \\
D_1 &= \sqrt{((9129.67 - 9130.80 \cdot \cos(324121/3600))^2 + (9130.80 \cdot \sin(324121/3600))^2)} \\
D_1 &= \sqrt{((9129.67 - 9130.80 \cdot \cos(90.033611))^2 + (9130.80 \cdot \sin(90.033611))^2)} \\
D_1 &= 12915.8889120 = 1.41454077539 \cdot 9130.80
\end{aligned}
$$

$$
\Delta(D_1) = 12916.3343421 - 12915.8889120 = 0.445430''
$$

$$
\begin{aligned}
D_2 &= \sqrt{((9123.97 - 9130.80 \cdot \cos(\angle(S,E)))^2 + (9130.80 \cdot \sin(\angle(S,E)))^2)} \\
D_2 &= \sqrt{((9123.97 - 9130.80 \cdot \cos(90 - 1'15'' - 5'21''))^2 + (9130.80 \cdot \sin(90 - 1'15'' - 5'21''))^2)} \\
D_2 &= \sqrt{((9123.97 - 9130.80 \cdot \cos(323604/3600))^2 + (9130.80 \cdot \sin(323604/3600))^2)} \\
D_2 &= \sqrt{((9123.97 - 9130.80 \cdot \cos(89.890000))^2 + (9130.80 \cdot \sin(89.890000))^2)} \\
D_2 &= 12895.6757575 = 1.41232704227 \cdot 9130.80
\end{aligned}
$$

$$
\begin{aligned}
D_2 &= \sqrt{((9129.67 - 9119.15 \cdot \cos(\angle(N,W)))^2 + (9119.15 \cdot \sin(\angle(N,W)))^2)} \\
D_2 &= \sqrt{((9129.67 - 9119.15 \cdot \cos(90 + 3'20'' - 7'33''))^2 + (9119.15 \cdot \sin(90 + 3'20'' - 7'33''))^2)} \\
D_2 &= \sqrt{((9129.67 - 9119.15 \cdot \cos(323747/3600))^2 + (9119.15 \cdot \sin(323747/3600))^2)} \\
D_2 &= \sqrt{((9129.67 - 9119.15 \cdot \cos(89.929722))^2 + (9119.15 \cdot \sin(89.929722))^2)} \\
D_2 &= 12895.9502905 = 1.41235710896 \cdot 9130.80
\end{aligned}
$$

$$
\Delta(D_2) = 12895.6757575 - 12895.9502905 = -0.274533''
$$

$$
: \quad 19.94'' < |D_1 - D_2| < 20.66''
$$

$$
D_1 > D_2
$$

We notice that the two ways of computing D_1 and D_2 produce slightly different results, $|\Delta(D_1)| = 0.34''$, and, $|\Delta(D_2)| = 0.45''$, for the raw Survey data; with, $|\Delta(D_1)| = 0.45''$, and, $|\Delta(D_2)| = 0.27''$, for the Corrected data. So, the errors are on the order of 1/2 inch. But, we shouldn't really expect any better here, since we've just seen that the "path" around the socket square itself is off by 1/2 inch. If the raw survey data can't close the path with an error better than 1/2 inch, we wouldn't expect the diagonals computed from that same data to be any more accurate than that same 1/2 inch variance, which is what we see. Our corrected data doesn't do that much better either, so it tells us that most of the error of survey is likely in the azimuth data, which is common to both calculations of the diagonal here. Of course, this $0.5''$ error, for a system whose scale is set by the length of the East side being taken as our reference unit, implies a relative error of, $err = 0.5''/9130.8'' \approx 0.00005 = 1/20,000$.

Figure 9: *The two-fingers hieroglyph.*

This brings up an interesting question; Since, the "two-finger" hieroglyph in the ancient Egyptian language has a dual meaning. On the one hand, the single finger glyph represents the number $10,000$; and when this glyph is repeated n times it denotes the number $n \times 10,000$. So, the "two-finger" hieroglyph means "both" $20,000$, and, as we've seen before, is used as the determinative in words like "accurate" and "precise." So, maybe the dual use is there to convey the idea that these Egyptian constructions are actually built to that level of precision, that is, 1 in 20,000 parts of a length, like here in the Great Pyramid? This would suggest that Petrie could not really do any better than he did, since the $0.5''$ error he thought must be due to earthquake movements and/or the settling of the land over the millennia, may just in fact be the result of the actual precision tolerance the architect chose to construct the monument in the first place.

$$\begin{aligned} \text{Survey data:} \quad &: \quad 19.90'' < |D_1 - D_2| < 20.69'' \\ \text{Corrected data:} \quad &: \quad 19.94'' < |D_1 - D_2| < 20.66'' \end{aligned}$$

We notice that the difference in the lengths of the two diagonals is on the order of 1 Royal cubit ($20.6''$), and the Corrected Data results show a slight tightening of the range about this value, as compared to the raw Survey Data. That makes us think the architect's 5th parameter to define the specific shape of the socket square quadrilateral could be that very difference, $D_1 - D_2 = 1\ cu$, instead of the diagonal crossing angle, Θ. But, as we mentioned, the *units* themselves are variable metrics, and only the *counts* have exact and precise values. So, we chalk this up to yet another mysterious synchronicity the architect manages to extract out of a rather complex problem tweaking the socket square, which is also the *reference square* in laying out the site for this monument.

Now the observation that the two diagonal lengths are close to each other, but differ from a well known characteristic length, of the Royal Cubit, suggests we explore the idea of these lengths forming yet another pair of *reference* and *apposite* values to encode another count. So we examine this idea.

$$n \quad = \quad 1/\sqrt{(1 - D_2/D_1)} \tag{148}$$

Raw Survey data:

$$n = \left(1 - \frac{D_2(\angle(S,E))}{D_1(\angle(S,W))}\right)^{-1/2} = \left(1 - \frac{12895.6263263}{12916.3201598}\right)^{-1/2} = 24.98 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(S,E))}{D_1(\angle(N,E))}\right)^{-1/2} = \left(1 - \frac{12895.6263263}{12915.9808574}\right)^{-1/2} = 25.19 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(N,W))}{D_1(\angle(N,E))}\right)^{-1/2} = \left(1 - \frac{12896.0775244}{12915.9808574}\right)^{-1/2} = 25.47 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(N,W))}{D_1(\angle(S,W))}\right)^{-1/2} = \left(1 - \frac{12896.0775244}{12916.3201598}\right)^{-1/2} = 25.26 \rightarrow 25$$

$$n = \left(1 - \frac{avg(D_2)}{avg(D_1)}\right)^{-1/2} = \left(1 - \frac{12895.8519254}{12916.1505086}\right)^{-1/2} = 25.23 \rightarrow 25$$

Corrected data:

$$n = \left(1 - \frac{D_2(\angle(S,E))}{D_1(\angle(S,W))}\right)^{-1/2} = \left(1 - \frac{12895.6757575}{12916.3343421}\right)^{-1/2} = 25.00 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(S,E))}{D_1(\angle(N,E))}\right)^{-1/2} = \left(1 - \frac{12895.6757575}{12915.8889120}\right)^{-1/2} = 25.28 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(N,W))}{D_1(\angle(N,E))}\right)^{-1/2} = \left(1 - \frac{12895.9502905}{12915.8889120}\right)^{-1/2} = 25.45 \rightarrow 25$$

$$n = \left(1 - \frac{D_2(\angle(N,W))}{D_1(\angle(S,W))}\right)^{-1/2} = \left(1 - \frac{12895.9502905}{12916.3343421}\right)^{-1/2} = 25.17 \rightarrow 25$$

$$n = \left(1 - \frac{avg(D_2)}{avg(D_1)}\right)^{-1/2} = \left(1 - \frac{12895.8130240}{12916.1116271}\right)^{-1/2} = 25.23 \rightarrow 25$$

Noticing that, $D_1 > D_2$, and assigning, $D = D_1$ and $D' = D_2$, with the usual, $\Delta D(25) = D - D'$, we test the fit to the theoretical specifications. First we evaluate the range requirements:

$$\frac{1}{2}\left(\frac{1}{26^2} + \frac{1}{25^2}\right) < \frac{\text{meas}\,\langle \Delta D(25)\rangle}{D} < \frac{1}{2}\left(\frac{1}{24^2} + \frac{1}{25^2}\right)$$

$$\frac{1301}{845000} < \frac{\text{meas}\,\langle \Delta D(25)\rangle}{D} < \frac{1201}{720000}$$

$$0.00153964497 < \frac{\text{meas}\,\langle \Delta D(25)\rangle}{D} < 0.00166805556$$

$$\frac{1}{649.500} < \frac{\text{meas}\,\langle \Delta D(25)\rangle}{D} < \frac{1}{599.500}$$

Then we pick the best numerical result, $n = 25.00$, which appears in the Corrected data, so, $D = 12916.3343421''$, and, $D' = 12895.6757575''$, to verify the fit:

$$\frac{D}{649.500} < \text{meas}\,\langle \Delta D(25)\rangle < \frac{D}{599.500} \tag{149}$$

$$\frac{12916.3343421}{649.500} < \text{meas}\,\langle \Delta D(25)\rangle < \frac{12916.3343421}{599.500} \tag{150}$$

$$19.886581 < \text{meas}\,\langle \Delta D(25)\rangle < 21.545178 \tag{151}$$

$$\text{meas}\,\langle \Delta D(25)\rangle = D - D' = 12916.3343421 - 12895.6757575 = 20.658585$$

$$19.886581 < 20.658585 < 21.545178 \ \checkmark \tag{152}$$

Now we pick the worst numerical result, $n = 25.47$, which appeares in the raw Survey data, so, $D = 12915.9808574''$, and, $D' = 12896.077524''$, to check the fit:

$$\frac{D}{649.500} < \text{meas}\,\langle \Delta D(25) \rangle < \frac{D}{599.500} \tag{153}$$

$$\frac{12915.9808574}{649.500} < \text{meas}\,\langle \Delta D(25) \rangle < \frac{12915.9808574}{599.500} \tag{154}$$

$$19.886037 < \text{meas}\,\langle \Delta D(25) \rangle < 21.544589 \tag{155}$$

$$\text{meas}\,\langle \Delta D(25) \rangle = D - D' = 12915.9808574 - 12896.077524 = 19.903333$$

$$19.886037 < 19.903333 < 21.544589 \ \checkmark \tag{156}$$

and we see even the worst of all possible *count* values, n, indicated from the various combinations of D_1 and D_2 still meet the theoretical specifications, which tells us that, $n = 25$, is unequivocally intended to be the number of divisions in the socket square's diagonal length. The *unit* of length implied in the diagonal is then on the order of $516.65''$.

$$D = 12916.3343421'' = n \cdot u = 25 \times 516.653374'' \tag{157}$$

But, we notice, $516.65''/20.632'' = 25.04 \ \rightarrow \ 25$, so the unit of length is about 25 royal cubits. We recall the King's Chamber also has a diagonal that is 25 cubits. The dimensions of the King's Chamber is, nominally, given by, $10(W) \times 20(L) \times 5\sqrt{5}(H)$, in cubit measures, so that the cubic diagonal of the room is, $\sqrt{10^2 + 20^2 + (5\sqrt{5})^2} = \sqrt{625} = 25$; and we can see that the socket square diagonal is then constructed of 25 of the King's Chamber cubic diagonal lengths. This also makes the socket square diagonal effectively, $25 \times 25 = 625$ royal cubits in length.

$$\begin{array}{lcll} D & = & n \cdot u & = 25 \cdot 516.653374'' & = 12916.3344'' = 12916.3343'' + 0.0001'' \\ D' & = & (n - 1/n) \cdot u & = (25 - 1/25) \cdot 516.653374'' & = 12895.6682'' = 12895.6758'' - 0.0076'' \end{array}$$

A curious observation here, is that, $12895.6757575''/625 = 20.633'' \approx 20.632''$, which is Petrie's Royal Cubit in the King's Chamber. But, $12916.3343421''/625 = 20.666''$, which is a little bit high. Yet, $12916.3343421''/626 = 20.633''$, giving us the Petrie result again. Given that, $D - D' \approx 1\ cu$, it seems that we should have, $D = 626 \cdot 1\ cu = 25 \cdot (25 + 1/25) \cdot 1\ cu$, and, $D' = 625 \cdot 1\ cu = 25 \cdot 25 \cdot 1\ cu$, instead of, $D = 625 \cdot\ 1\ cu = 25 \cdot 25 \cdot\ 1\ cu$, and, $D' = 624 \cdot\ 1\ cu = 25 \cdot (25 - 1/25) \cdot\ 1\ cu$. It appears that the architect may be favoring the '+' sign in the general reciprocal adjustment, $(n \pm 1/n)$, here, instead of the usual '$-$' sign.

Of course, when we change the 'sign' in the reciprocal adjustment, $(n \pm 1/n)$, the *count* remains the same, but the *unit* is now different.

$$D' = 12895.6757575'' = n \cdot u = 25 \times 515.827030'' \tag{158}$$

So, here we have, $u = 515.827030''$, vs the previous, $516.653374''$, and our parameters fit the data accordingly;

$$\begin{array}{lllll} D & = & (n + 1/n) \cdot u & = (25 + 1/25) \cdot 515.827030'' & = 12916.3088'' = 12916.3343'' - 0.0255'' \\ D' & = & n \cdot u & = 25 \cdot 515.827030'' & = 12895.6758'' = 12895.6758'' + 0.0000'' \end{array}$$

Now, $u/25 = 515.827030''/25 = 20.633''$, vs, $u/25 = 516.653374''/25 = 20.666''$, and we have two competing values for the length of the cubit; although, the cubit values are often found anywhere in the range, $20.5'' < 1\ cu < 20.7''$, typically, so both are technically possibly correct. Yet, we tend to favor the value found in the King's Chamber, just because of how well constructed that room is, with obvious intention to deliver the message of *counts* and *units* to the highest precision in the structure. At any rate, it's an observation worth mentioning.

> *An intriguing observation is that with the Pyramid height to apex nominally being* **280** *royal cubits, and the socket square diagonal being here found to be* **625** *of these same royal cubits, we have the ratio of height to diagonal as,*
>
> $$\frac{height}{diagonal} = \frac{280\,cu}{625\,cu} \equiv \frac{448\,cu}{1000\,cu}$$
>
> *but the* **448** *cu is the limit square and the number of divisions on the standard Royal Cubit Rod! And the mystery synchronicity of the repeating reference counts continues! Recall, the socket square and the apex height are just reference parameters used in the design layout, and the pyramid isn't actually built out to these extreme limits.*

Slightly different results are expected if we pair the reference and apposite, (D, D'), with a different pair, (D_1, D_2), from the various alternatives above. But, the essential conclusion is the same, that the King's Chamber cubit and cubic diagonal fit better with a socket square diagonal evaluation done with the positive sign in the reciprocal adjustment. This observation probably merits further investigation.

Summary of the Socket Square Results.

In summary, we find from the application of *the principle of reciprocal adjustment*, to the Socket Square length data from Petrie's survey, that the socket square is nominally a square with two division counts, 28×28 and 40×40, with longer length units, $s' = 10s$, and $s'' = 7s$. The simultaneous encoding of these two systems then suggest that smaller unit, s, for a count of 280×280.

As suspected, the cubit rod is not used for setting the lengths around the base here, but much longer lengths are used, in order to encode the reciprocal adjustments in a manner that make them discoverable through re-measurement. The two counts then suggest that smaller unit, s, which we have identified as typical of a man's step length, and two values, one rough and one precise, $s = 32.593''$ and $s_o = 32.608''$, are derived from the path length formulas; with the application of these units to the path length around the pyramid base seeming to explain the well known "$17.5''$ *West Displacement*" discrepancy in the South Lengths caused by the two reference points in the South West socket, as shown in Davidson's Plate XX.

While the King's Chamber units are derived from various hand metrics, the Socket Square *perimeter* units are derived from several foot metrics. We have the "*footstep*," "*seven footsteps*," and "*ten footsteps*," as our three primary lengths that measure the base square on the ground. These units are then found to be exactly that which define the *width* and *length* of the subterranean chamber, below the ground, i.e. $s' \times (s' + s'') = 10s \times (10s + 7s)$, validating the discovery of the units in the foundation socket square above it on the ground plan.

An examination of Petrie's azimuth data, at first, suggests *the principle of reciprocal adjustment* may also be used on angles of the diagonal crossing, with an '*implicit*' reference, $90 \cdot u$, and explicit apposite, $(90 - 1/90) \cdot u$, that is measured with a unit convention, $u = 1°$; making the circle division of $360°$ a possible encoded reference. However, a closer geometrical evaluation reveals that Petrie's particular survey data only allows the cross angle, Θ, to be in the approximate range, $90 - 1/2 < \Theta < 90 - 1/87$, so that the target metric, $(90 - 1/90)$, falls just outside that range, reducing the observation to more of a suggestive *hint*, than a definite conclusion. Then, looking elsewhere for possible *checks* on this hint, we discover that by applying *The Reciprocal Theory* to *adjacent apposites*, we could also identify another encoded angle metric, and provided we recognise check equation, $28 + 40 + M/2 + M = 203$, as indicated, then the pair of adjacent lengths, (E, N), which subtend a nominal right angle, encode between them the count, $n = 90$, allowing for the correction of the data values, elevating $.1''$ precisions to $.01''$; and seeming to corroborate the circle number 360.

Finally, the pair of diagonals on this socket square, considered as another metrical pair, $(D, D') = (D_1, D_2)$, unequivocally encode the division count,

$n = 25$, with the difference in diagonal lengths being about 1 Royal Cubit, $D_1 - D_2 = 1\,cu$. But, in this case the positive sign in the reciprocal adjustment, $(n \pm 1/n)$, seems to better reflect the fit if the diagonal numbers are required to more closely match with the *cubit* and *cubic diagonal* numbers in the King's Chamber. It turns out that the socket square diagonal is 25 lengths of the unit taken as the length of the King's Chamber cubic diagonal. Once again, the use of *repetition* in the metrical parameters suggest correctness and intention by the architect of this monument.

5 Conclusion

Perhaps singularly characteristic in the surviving recorded works of Ancient Egyptians is their mysterious preoccupation with "*unit fractions*" in their mathematical art. They did have a few fraction forms like, 2/3 and 3/4, which they understood and gave special hieroglyphs for, so it was certainly not a matter of an inability to comprehend the idea of a fraction composed of two integers, one above and the other below, in representing the idea of a part of some whole number. Rather, it seems they deliberately *chose* to use this somewhat cumbersome method of expressing fractional numbers as a series composed of the summation of unique and distinct *reciprocals* of integers instead. Unique, in that the same reciprocal was not allowed to be repeated in the summation expression that represented the fraction. This appears to be some kind of code—a ritual established in the mathematical art intended to record perhaps some idea about the physical world.

Now, here in the Great Pyramid, we find, by application of a reciprocal adjustment—which is now obviously understood to be fully within the scope and thought patterns of the Egyptians with their preoccupation with these very same *unit fractions* in their art—a metrological means of encoding and extracting back out the metrical parameters used in design. In the King's Chamber, we find the two primary counts, 28 and 40, and in the Socket Square at the base of the pyramid, these two numbers, 28 and 40, appear once again. This simply cannot be a coincidence, for the numerical counts not only appear repeatedly, but in the exact orientations in and among the four cardinal directions.

On the one hand, the King's Chamber is indicated to be a room with the two dimensions, 20 and 40; the length, L, being obviously twice the width, W, on simple inspection. While, the squares at the base of the pyramid can be thought of as having again two usually related lengths: a displacement, L, used in the normal calculations by the designer of the slope of the monument, as through the *seked* metric, $s = L/H$; and the distance between two corners along the length of one side, $S = 2L$. So, here again, we have the "*seked length*" and the

"*side length*," the two related metrics that are usually involved in the design, one twice the other, suggestively partitioned by the numbers, 20 and 40, being obviously divided here by the same two counts as used in the King's Chamber. But, when we divide a length into n equal parts, that corresponding *reciprocal*, or, if we prefer, *unit fraction*, $1/n$, is now the *resolution* of the scale that takes that length as the metric range of a unit of length.

Then, on the other hand, the King's Chamber is also indicated to be a room with the two dimensions, 28 and 56; and the perimeter of this room then has count, 168, which is the number of 'digit' divisions on the unit of the measuring rod called the *reed*, which is 6 Royal Cubits long (Ezekiel 40:5), and also the number of hours in a week of 7 days; That first number, 28, being one of the *Perfect Numbers*, that so fascinated the Greeks. The architect here is obviously also fascinated by this number, 28, as he set the truncated height of the Great Pyramid such that the square of the time a stone falls to the ground from that truncated platform is $28\,s^2$, and the *second* is the heart beat of a man. Man also has 28 finger segments on 10 fingers, and the height of the Great Pyramid to apex is set nominally to $28 \times 10 = 280$ cubits, or forearm lengths. Man also has 28 segments on his 10 toes, and again the architect sets the socket square side length to $28 \times 10 = 280$ footsteps.

At any rate, from the obvious *repetition* in certain numeric counts, and the seemingly possible explanation that might exist to explain *why* those counts, the conclusion we inevitably arrive at is that the ancient Egyptians must have intentionally put these numbers into this monument, and must therefore have used this *principle of reciprocal adjustment* in the construction. It is possible that they employed both signs, $(n \pm 1/n)$, in the construction, here and there, depending on what was required to fit the dimensions, as suggested in the diagonals of the socket square; and they may have used other obvious variations elsewhere, such as, $L = (n + 1/n) \cdot u$, with apposite, $L' = (n - 1/n) \cdot u$, with the true metric intended to be the *mean* between these two slightly different values, $\langle L \rangle = (L + L')/2$, as suggested by the quote from J. Ralston Skinner in his text. But, in our text here we selected the '$-$' sign variant, $(n - 1/n)$, because it seemed to logically follow from the ancient method of *anthyphairesis* which is known to have been used to measure things in the past, and we have only looked at two places so far in Egyptian monuments with this type of reciprocal system; (1) the King's Chamber and (2) the Socket Square, of the Great Pyramid. Our sample set of data is too small to make any inference about whether or not this was a general method used by the Egyptians in all their designs, or whether it is singularly unique to The Great Pyramid alone. We only have the 1664 A.D. quote from *F. Vauslab* that he had "*taken notice that none of the Pyramids' (bases) are alike, or perfectly square; but that all have two sides longer than the others*," and Skinner's quote of Prof Piazzi Smyth's observation that such variances were to be found all over the place, to suggest this might be a general theme in the ancient Egyptian architecture, not restricted to the Great Pyramid alone, but that it has simply gone unexplored to date.

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6 APPENDIX A - Precision Analysis

King's Chamber

Precision: $0.01''$

	N	E	S	W
Top	412.1449	206.3049	411.8849	206.0449
	412.14	206.30	411.88	206.04
	412.1350	206.2950	411.8750	206.0350
Base	412.7849	206.4349	412.5349	206.1649
	412.78	206.43	412.53	206.16
	412.7750	206.4250	412.5250	206.1550

A	Top:	$max(\Delta L(40))$	$= 412.1449 - 411.8750 = 0.2699$	$\rightarrow$	$0.251388 < 0.2699 \not< 0.264283$ x
B	Top:	$min(\Delta L(40))$	$= 412.1350 - 411.8849 = 0.2501$	$\rightarrow$	$0.251378 \not< 0.2501 < 0.264274$ x
C	Top:	$max(\Delta W(28))$	$= 206.3049 - 206.0350 = 0.2699$	$\rightarrow$	$0.254227 < 0.2699 < 0.273071$ ✓
D	Top:	$min(\Delta W(28))$	$= 206.2950 - 206.0449 = 0.2501$	$\rightarrow$	$0.254214 \not< 0.2501 < 0.273058$ x
E	Base:	$max(\Delta L(40))$	$= 412.7849 - 412.5250 = 0.2599$	$\rightarrow$	$0.251775 < 0.2599 < 0.264691$ ✓
F	Base:	$min(\Delta L(40))$	$= 412.7750 - 412.5349 = 0.2401$	$\rightarrow$	$0.251769 \not< 0.2401 < 0.264684$ x
G	Base:	$max(\Delta W(28))$	$= 206.4349 - 206.1550 = 0.2799$	$\rightarrow$	$0.254387 < 0.2799 \not< 0.273243$ x
H	Base:	$min(\Delta W(28))$	$= 206.4250 - 206.1649 = 0.2601$	$\rightarrow$	$0.254375 < 0.2601 < 0.273230$ ✓

Precision: $0.001''$

	N	E	S	W
Top	412.14049	206.30049	411.88049	206.04049
	412.140	206.300	411.880	206.040
	412.13950	206.29950	411.87950	206.03950
Base	412.78049	206.43049	412.53049	206.16049
	412.780	206.430	412.530	206.160
	412.77950	206.42950	412.52950	206.15950

I	Top:	$max(\Delta L(40))$	$= 412.14049 - 411.87950 = 0.26099$	$\rightarrow$	$0.251381 < 0.26099 < 0.264277$ ✓
J	Top:	$min(\Delta L(40))$	$= 412.13950 - 411.88049 = 0.25901$	$\rightarrow$	$0.251381 < 0.25901 < 0.264277$ ✓
K	Top:	$max(\Delta W(28))$	$= 206.30049 - 206.03950 = 0.26099$	$\rightarrow$	$0.254221 < 0.26099 < 0.273065$ ✓
L	Top:	$min(\Delta W(28))$	$= 206.29950 - 206.04049 = 0.25901$	$\rightarrow$	$0.254220 < 0.25901 < 0.273064$ ✓
M	Base:	$max(\Delta L(40))$	$= 412.78049 - 412.52950 = 0.25099$	$\rightarrow$	$0.251772 \not< 0.25099 < 0.264688$ x
N	Base:	$min(\Delta L(40))$	$= 412.77950 - 412.53049 = 0.24901$	$\rightarrow$	$0.251772 \not< 0.24901 < 0.264687$ x
O	Base:	$max(\Delta W(28))$	$= 206.43049 - 206.15950 = 0.27099$	$\rightarrow$	$0.254381 < 0.27099 < 0.273237$ ✓
P	Base:	$min(\Delta W(28))$	$= 206.42950 - 206.16049 = 0.26901$	$\rightarrow$	$0.254380 < 0.26901 < 0.273236$ ✓

Precision: $0.01''$

Given that Petrie's reported precision is only to one-hundredth's of an inch, that is, $0.01''$, the true measures of the lengths and widths could very well be somewhat higher or lower than the stated values. For example, $L = 412.14''$, could actually be anywhere in the range, $412.1350'' < L < 412.1449''$, and all the possible values in this range would still be reported as, $L = 412.14''$. In the top table, we list all Petrie's values along with the higher possible number above, and lower possible number below, for the top set and the base set measures of the rectangles on the ceiling and the floor.

Then, we select those values that would give us the max Δ, and min Δ, for the length, $L(40)$, and the width, $W(28)$, of the ceiling—rows A,B,C,D—and the floor—rows E,F,G,H. And then, taking the larger value for our reference, as usual, we investigate if the measured Δ satisfies the specification range, using the previously derived theoretical range formulas:

$$\frac{L}{1639.500} < \text{meas}\,\langle \Delta L(40) \rangle < \frac{L}{1559.500} \tag{49}$$

$$\frac{W}{811.500} < \text{meas}\,\langle \Delta W(28) \rangle < \frac{W}{755.500} \tag{50}$$

Looking at the table, we see that row 'A' falls out of specs, and there are some values for the length pair, L, L', that would indicate the count, $n = 39$, instead of our anticipated, $n = 40$. Row 'B' has a few values of this pair yielding the count, $n = 41$, instead of our, $n = 40$. For the width pair, W, W', row 'C' is within spec, $n = 28$. But, row 'D' falls out of spec, indicating the count, $n = 29$, instead of, $n = 28$. So, there is a scenario, where the round-off error could represent the data for the ceiling as meeting all the specifications of *The Reciprocal Theory*, whereas, in fact, none of the 'true' measures actually falls within the specification range! Similarly, for the floor rectangle, L, L', row 'E' is within spec, agreeing with count, $n = 40$. But, row 'F' falls out of spec, suggesting, $n = 41$. Then, for the widths, W, W', row 'G' falls out of spec, with a few values yielding, $n = 27$, instead of, $n = 28$. Finally, row 'H' is within spec, with the expected, $n = 28$. So, here again, there's a scenario where all the true measures of floor data actually contradict the hypothesis of the *The Reciprocal Theory* giving the results $n = 40$, for the length, and, $n = 28$, for the width; suggesting some nearby values are more apt to the fit.

Precision: $0.001''$

Things change dramatically, however, if we just had one more digit after the decimal place. Petrie reports that while his measuring rule was only graduated to one-hundredth's of an inch, $0.01''$, those graduation marks were known to an accuracy of one-thousandths of an inch, $0.001''$. If we assumed that Petrie's data were, in fact, accurate to $1/1000$ of an inch, and so added a $'0'$ to the data he reports to reflect this, and ran the analysis again, we end up with the second table above. Now, for all possible values of the round-off error, the ceiling–rows I, J, K, L–are always within spec, regardless of how those numbers are rounded to produce the reported data. Similarly, the 'width' of floor–rows O, P–is again always within spec. However, there's still a problem with the length pair, L, L', at the floor level, as there are scenarios where the suggested count here is, $n = 41$, instead of, $n = 40$. So, again, it seems there's a problem with Petrie's data for the lengths of the room on the floor rectangle, and this problem does not go away even if we arbitrarily increase the precision of his data synthetically as we have done here for the rest of his data values.

This suggests that one or the other of Petrie's floor lengths, $L = 412.78''$ or $L' = 412.53''$, is off. But there are scenarios that his numbers could be correct to the precision reported, just that it requires one more significant figure to the right of the decimal to see it. For example, if the short data point were, $412.526'' \rightarrow 412.53''$, and we then synthetically increased the precision to the 3rd decimal place for all values as before, we get the following table.

Precision: $0.001''*$

	N	E	S	W
Top	412.14	206.30	411.88	206.04
	412.140	206.300	411.880	206.040
Base	412.78	206.43	412.53	206.16
	412.780	206.430	412.526	206.160

Q	Top:	meas($\Delta L(40)$)	$= 412.140 - 411.880 = 0.260$	$\rightarrow$	$0.251382 < 0.260 < 0.264277$	✓
R	Top:	meas($\Delta W(28)$)	$= 206.300 - 206.040 = 0.260$	$\rightarrow$	$0.254221 < 0.260 < 0.273064$	✓
S	Base:	meas($\Delta L(40)$)	$= 412.780 - 412.526 = 0.254$	$\rightarrow$	$0.251772 < 0.254 < 0.264687$	✓
T	Base:	meas($\Delta W(28)$)	$= 206.430 - 206.160 = 0.270$	$\rightarrow$	$0.254381 < 0.270 < 0.273236$	✓

	N	E	S	W
	412.78049	206.43049	412.52649	206.16049
Base	412.780	206.430	412.526	206.160
	412.77950	206.42950	412.52550	206.15950

U	Base:	$max(\Delta L(40))$	$= 412.78049 - 412.52550 = 0.25499$	$\rightarrow$	$0.251772 < 0.25499 < 0.264688$	✓
V	Base:	$min(\Delta L(40))$	$= 412.77950 - 412.52649 = 0.25301$	$\rightarrow$	$0.251772 < 0.25301 < 0.264687$	✓

So, all the lengths and widths of the room are now in full agreement with *The Reciprocal Theory*, and yield the values of, $n = 40$, for the length, and, $n = 28$, for the width. With this one slight change, consistent with the reported

data precision, the rows Q, R, S, T, all reveal this particular data set fits the specifications required. And now, even when we examine the max Δ, and min Δ, for the extremes of the subsequent round-off ranges to look for violations of the specs, we find all the pre-rounded true data possible for this data set are compliant and fall within the theoretical specification range. Everything fits as would be expected.

Considering that just one slight change to one datum brings the whole survey data set into compliance, supporting *The Reciprocal Theory*, that yields the counts, 28 and 40; while it would take many changes to the data, otherwise, to completely reject the theory, the odds are that the theory is correct. But, it still must be emphasized that this is not absolute proof, since weird things can happen, as we have seen in the tables above—a confluence of accidental coincidences in round-off can create the illusion of fit, where there is none. Only re-measuring the room to one order of magnitude more, $0.001''$, could convince the most ardent critic.

Socket Square

Precision: $0.1''$

	N	E	S	W
	9129.849	9130.849	9123.949	9119.249
Socket	9129.8	9130.8	9123.9	9119.2
	9129.750	9130.750	9123.850	9119.150

A	Soc:	$max(\Delta L(40))$	$= 9129.849 - 9123.850 = 5.999$	$\rightarrow$	$5.568679 < 5.999 \not< 5.854344$	x
B	Soc:	$min(\Delta L(40))$	$= 9129.750 - 9123.949 = 5.801$	$\rightarrow$	$5.568618 < 5.801 < 5.854280$	✓
C	Soc:	$max(\Delta W(28))$	$= 9130.849 - 9119.150 = 11.699$	$\rightarrow$	$11.251816 < 11.699 < 12.085836$	✓
D	Soc:	$min(\Delta W(28))$	$= 9130.750 - 9119.249 = 11.501$	$\rightarrow$	$11.251694 < 11.501 < 12.085705$	✓

Precision: $0.1''$

When it comes to the Socket Square, if we take Petrie's survey data for the lengths to one decimal place as given, $0.1''$, and explore the round-off range that's consistent with the reported values, we find the East/West parallels in full agreement with the specifications. So, rows 'C', and 'D', here, that describe the lengths differing by about $11.5''$ receive our red checkmark ✓ of approval. So, we're confident in our count, $n = 28$, for the East side of the square. However, the North/South pair with length difference about $5.8''$ has a problem, for while row 'B' for the min Δ agrees with, $n = 40$, row 'A' for max Δ shows there are some length pairs, L, L', that suggest, $n = 39$, to be the count. We saw this

when discovering the count of the mid data before, and explained why we think this is an error, since we notice a 'repetition' of themes from the King's Chamber being used here on the Socket Square; and also the count, $n = 40$, when chosen, is seen validated by the measurements in the *subterranean chamber*. But, we notice that Petrie's data could be considered good, as is, if we just changed two values to suitable pre-rounding state.

Precision: $0.01''$

	N	E	S	W
Socket	9129.8	9130.8	9123.9	9119.2
$\pm.03$	9129.77	9130.80	9123.93	9119.20

E	Soc:	meas $\langle \Delta L(40) \rangle$	$= 9129.77 - 9123.93 = 5.84$	$\rightarrow$	$5.568631 < 5.84 < 5.854293$	✓
F	Soc:	meas $\langle \Delta W(28) \rangle$	$= 9130.80 - 9119.20 = 11.60$	$\rightarrow$	$11.251756 < 11.60 < 12.085771$	✓

Precision: $0.01''$

	N	E	S	W
	9129.7749	9130.8049	9123.9349	9119.2049
Socket	9129.77	9130.80	9123.93	9119.20
	9129.7650	9130.7950	9123.9250	9119.1950

G	Soc:	$max(\Delta L(40))$	$= 9129.7749 - 9123.9250 = 5.8499$	$\rightarrow$	$5.568634 < 5.8499 < 5.854296$	✓
H	Soc:	$min(\Delta L(40))$	$= 9129.7650 - 9123.9349 = 5.8301$	$\rightarrow$	$5.568628 < 5.8301 < 5.854290$	✓
I	Soc:	$max(\Delta W(28))$	$= 9130.8049 - 9119.1950 = 11.6099$	$\rightarrow$	$11.251762 < 11.6099 < 12.085777$	✓
J	Soc:	$min(\Delta W(28))$	$= 9130.7950 - 9119.2049 = 11.5901$	$\rightarrow$	$11.251750 < 11.5901 < 12.085764$	✓

Precision: $0.01''$

This prompts us to look at the case where we synthetically upgrade the East/West data to one more decimal, and do the same for North/South, but just apply a small $\pm 0.03''$ adjustment to the raw data to explain the drift that occurs making things out-of-spec by rounding. Now, rows 'E' and 'F' show that the counts, $n = 40$, and, $n = 28$, are selected, and all pre-rounded values in the subsequent range, as rows 'G', 'H', 'I', 'J', demonstrate, fall within the theoretical specifications.

Now, we showed before, that by *backing-out* the data for the North and South values, we could ideally increase the South data by $.2''$, i.e., $9123.9'' + 0.2'' = 9124.1''$, or, alternatively, we could decrease the North data by $.2''$, i.e., $9129.8'' - 0.2'' = 9129.6''$, and, either way, we'd find the data now fits the count, $n = 40$, which we expect. We also pointed out that this was within Petrie's given possible error, $\pm 0.3''$, as the adjustment required was well within range, $0.2'' < | \pm 0.3''|$. But, while that would be the "ideal" situation, putting the count securely "at" the required count, $n = 40$, we only really need the data to refer to this count by a calculation that gets closer to it than to its neigbors. As we can see here, a tenth of the adjustment, $\pm 0.03''$, is all it takes to shift the value close enough to the count, $n = 40$, for that one to be selected.

At any rate, if it were possible to re-measure the Socket Square to 1/100 of an inch, the most ardent critic could be appeased. Unfortunately, nobody thought to protect these rock cutouts from the time they were uncovered, so they have been gradually deteriorating over time with exposure to the elements of weather, tourist foot traffic, and all the camels and vehicles moving about the Giza plateau, disregarding the sacred features around the pyramid. So today, we are at least lucky, to have Petrie's data, as he arrived at Giza just in the nick of time, not long after the sockets were uncovered from all the rubble, and before the industrial revolution created the modern transportation systems that bring large numbers of tourists and others to the plateau to witness the marvels. Hence, Petrie's data will have to do. They are, in fact, pretty good as is, and despite the small problems here and there with the precision used, with the corroborating evidence from other parts of the monument, it seems we can successfully re-construct some parts of the message the architect intended.